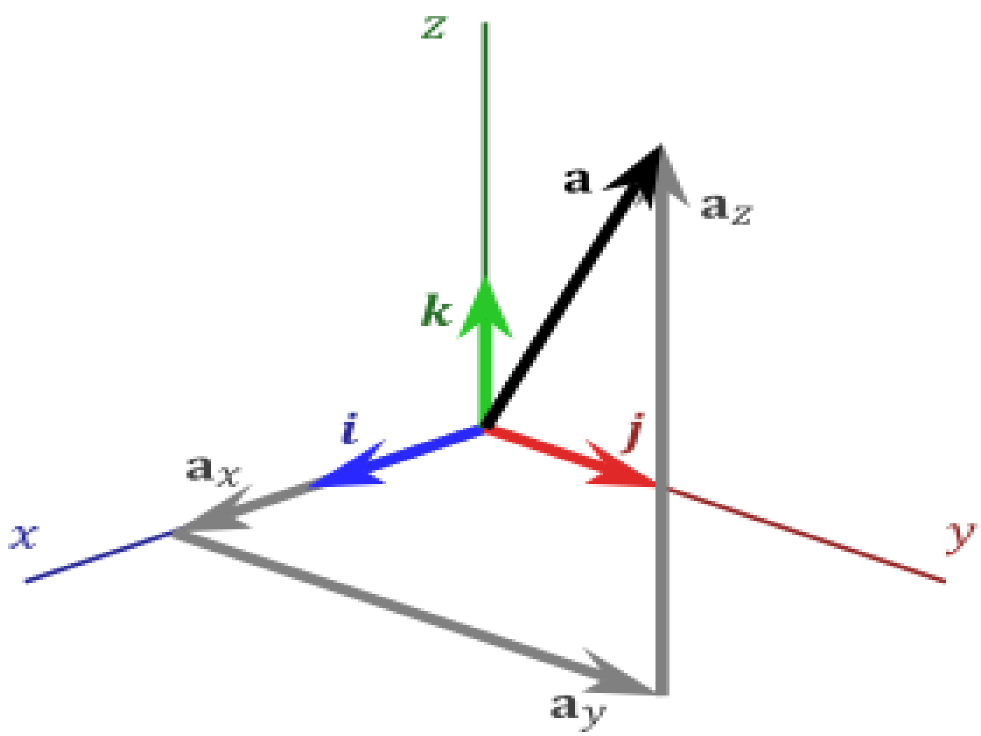
Physics C ***ultimate*** review packet created by NeighborhoodGeeks.

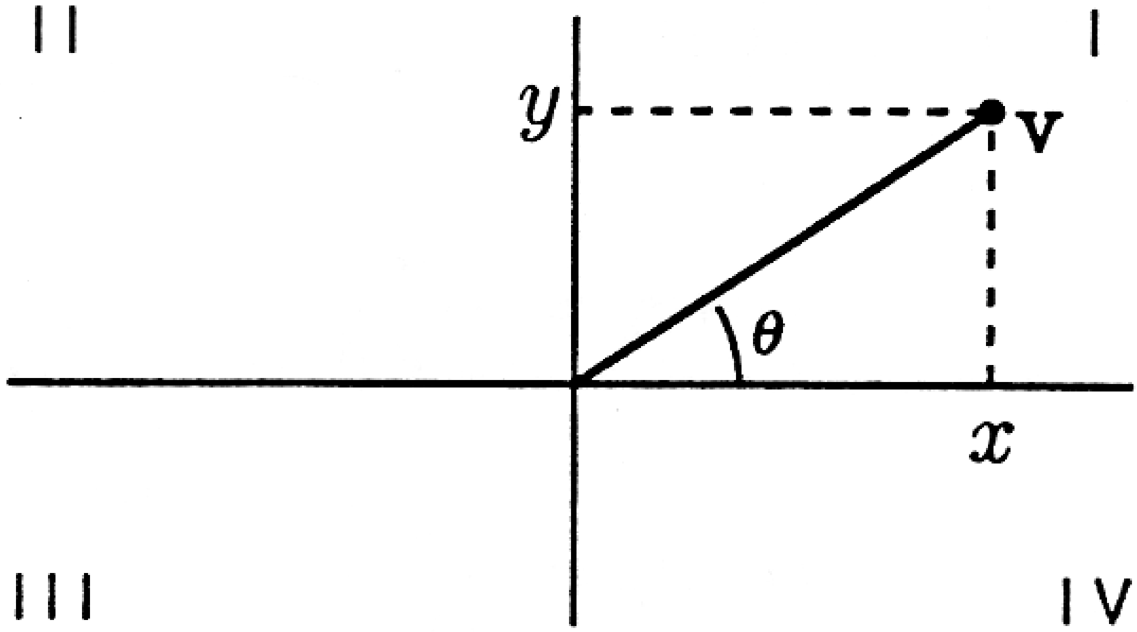
Unit 1: Vectors



The basics:

What is a vector?

A vector essentially is a line that connects two points, it has a distance or magnitude, and it has a direction.



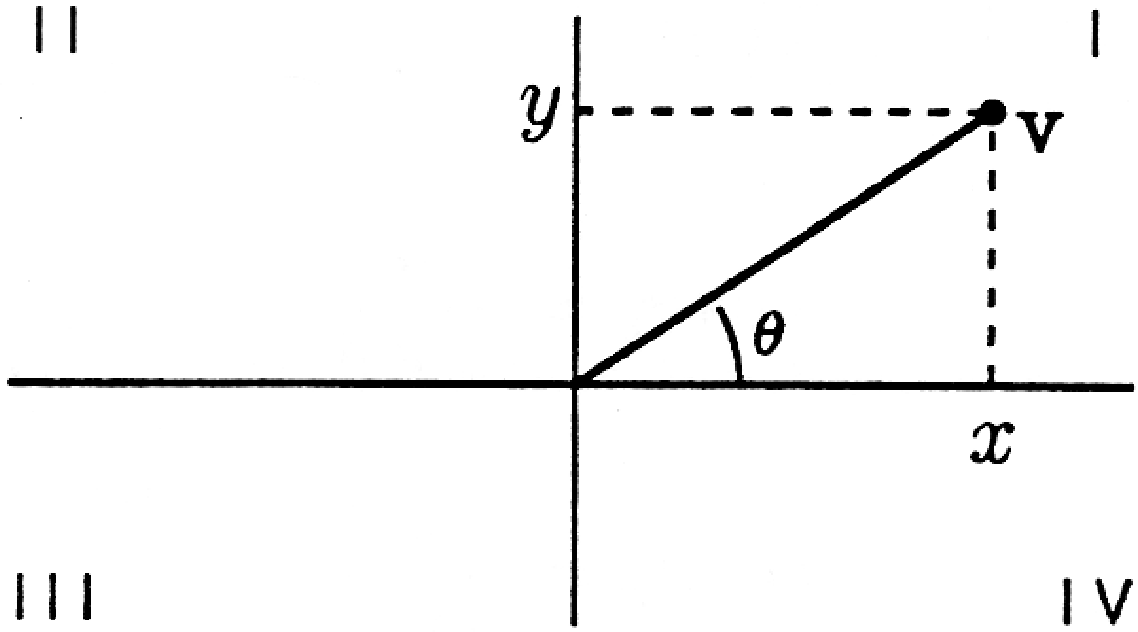
The above figure displays a vector, v. This vector has both x and y components. The x component of the vector tells us how far left or right the vector is, while the y component tells us how far up or down the vector is. For future reference, we shall declare that the vector extends from the origin (0,0) to the point (x,y).

Now in algebra II or even geometry, a question they could've asked you is to find the distance between the points (0,0) and (x,y) and you would use the distance formula,

distance = $\sqrt{(x-0)^2 + (y-0)^2}$

Depending on the teacher you had, you may not realize where that formula comes from, but regardless, I'm going to explain the derivation of the formula right now.

Recall that the X and Y axis are always formed such that they are perpendicular to one another, in other words, when they intersect, they form a 90 degree angle. With that in mind, look at the picture of the vector again,



Notice how the side opposite the θ hits the x-axis such that a 90 degree angle forms. Therefore, what we are actually looking at is a right triangle. This right triangle in question has legs that have lengths, x, which is the length of the leg that is colinear with the x-axis, and another leg that has length y, which is the length of the leg that is parallel to the y-axis.

The length of the vector is the hypotenuse of the right triangle formed by the two legs because it is the side of the right triangle that is opposite that of the 90 degree angle.

Well that's very nice, because we know how to calculate the length of the hypotenuse, that's just Pythagorean Theorem!!

Recall that Pythagorean Theorem states that

$C^2 = a^2 + b^2$

Where C is the length of the hypotenuse, and, in this case, the length of the vector, V, and a and b represent the lengths of the legs. Because we know the length of the legs are x and y respectively, we can solve for the length of the vector by isolating C.

$C = \sqrt{a^2 + b^2}$

To denote the length of vector V, mathematicians use absolute value bars.

Therefore, the length of vector V is denoted as $|V|$

Plugging into good ol' pythag, we get the basic equation

$$|V| = \sqrt{x^2 + y^2}$$

Since we know that the vector, V has both x and y components, what in reality is happening is that we are adding the vector, X which extends from (0,0) to (x,0) to the vector, Y which extends from (0,0) to (0,y).

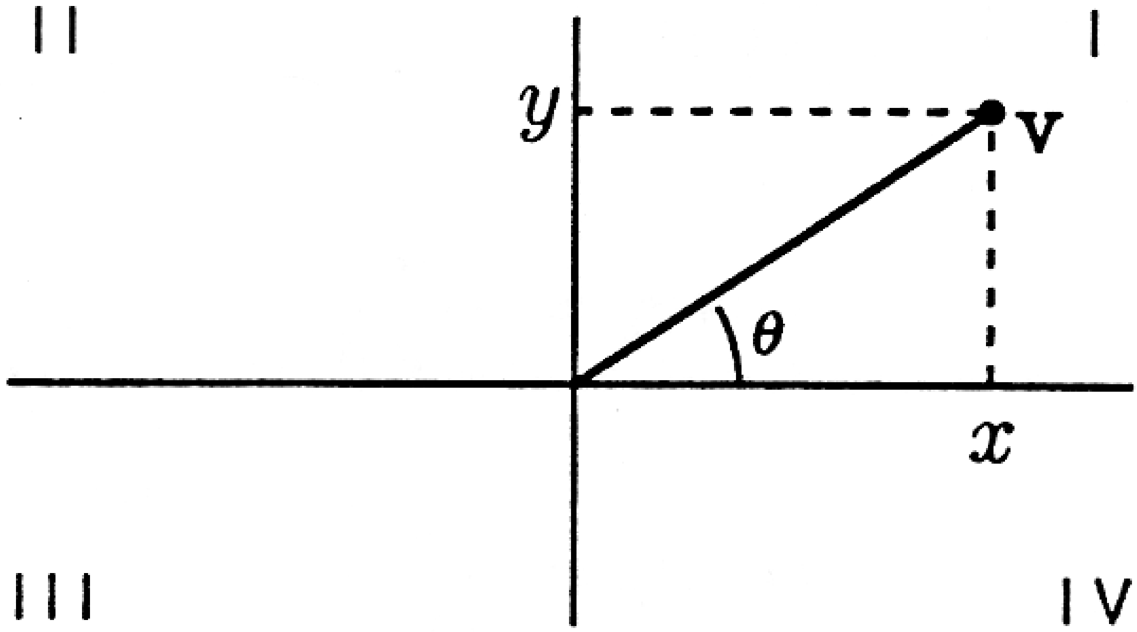
Now let's talk about finding individual components of vectors since this will be of use to us in physics.

Recall the following formulas:

$$sin\theta = \frac{Opposite}{Hypotenuse}$$

$$cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$tan\theta = \frac{Opposite}{Adjacent}$$



Using trig functions, we can find out the individual components of the vector in question.

We know that the magnitude of the vector V is equal to the hypotenuse of the right triangle. For the remainder of this lesson, we're going to refer to the x-component of V as V_x and likewise, the y-component of V will be referred to as V_y.

Recall $\sin\theta = \frac{Opposite}{Hypotenuse}$

Well the opposite side of the angle θ is going to be how far up the vector is, therefore it is going to be V_y.

Plugging into the above equation yields the following results:

$\sin\theta = \frac{V_y}{|V|}$

Isolating V_y we get the basic equation

$|V|\sin\theta = V_y$

To get the x-component of the vector V we can do a similar process, except now we will use cosθ.

Recall that $\cos\theta = \frac{Adjacent}{Hypotenuse}$

Well the adjacent side to the angle θ is how far the vector V is to the right, therefore it represents the x-component of the vector, or V_x

Plugging into the above equation yields the following results:

$cos\theta = \frac{V_x}{|V|}$

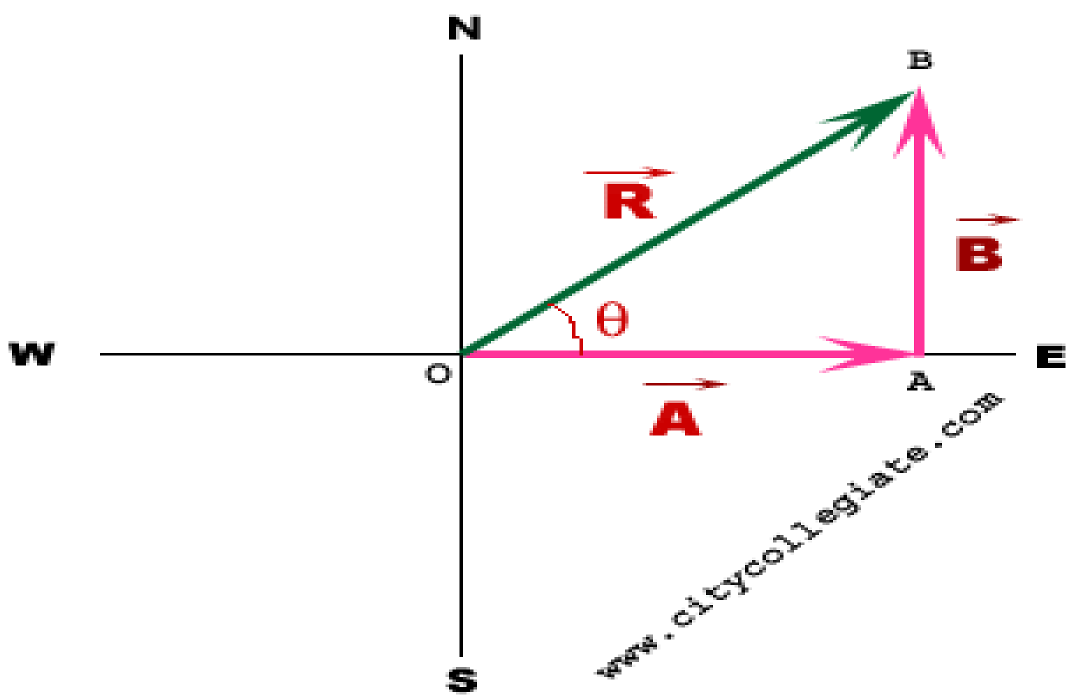
Isolating V_x gets the basic equation

$|V|\cos\theta = V_x$

Note that ***this is only because the angle θ is with respect to the x-axis***, if the angle θ was with respect to the y-axis then the sin and cos will switch. Usually the problem will give us an angle that is with respect to the horizontal, however, this is not always the case, and thus you should do this component separation piecemeal all the time.

How to add vectors:

It’s very simple, it's through a process called the head-to-tail method, whereby you put the head of one and attach it to the tail of the other, the resultant is the line that connects it. Look at the below picture. R is the vector addition/resultant of vector A and vector B.



How to subtract vectors:

The same as adding vectors except you take the reflection of one of the vectors and add that way, you’re basically adding a negative. A - B = A + -B.

Dot product:

The dot product is a vector operator that determines the degree of "likeness", or in other words, how much vector A is in the direction of vector B.

Important things to note about the dot product:

1. The result of the dot product operator is a ***SCALAR*** quantity, meaning it does not have a direction.
2. A dot B = B dot A, in other words, dot product has a ***commutative property.***
3. If A dot B evaluates to 0, the two vectors are said to be ***orthogonal***, or ***perpendicular,*** we will see why this is so when we look at how to compute vectors.

How to compute the dot product of two vectors, A and B:

There are two ways to do this depending on the information given to you

A dot B = $|A|\ |B| \cos\theta$

A dot B = $\Sigma_i\, a_i b_i$

Remember how we said that if A dot B equals 0 then the two vectors are orthogonal, well $\cos(\pi/2) = 0$, hence if there is a 90 degree angle between the two vectors the dot product returns a value of 0.

Cool things that you can do with the dot product operator:

Find the angle between the two vectors
Find out if vectors are in the same plane

$|A|\ |B| \cos\theta = \Sigma_i\, a_i b_i$

Therefore,

$\mathrm{Cos}\theta = (\Sigma_i\, a_i b_i)/|A|\ |B|$

Therefore,

$\theta = \cos^{-1}([\Sigma_i\, a_i b_i]/|A|\ |B|\ )$

Examples of the dot product:

Lets say we have two vectors, A and B

$A = <1,2,3>$
$B = <-1,2,-3>$

In this example, we don't know the angle between the two vectors, so let's find it.

Recall previous equation,

$\theta = \cos^{-1}([\Sigma_i a_i b_i]/|A|\ |B|)$

Well although that looks real scary, it really isn't that hard to figure out, so let's take it piecemeal.

First order of business, what is $\Sigma_i a_i b_i$?

This scary notation just says sum up the corresponding components of vector A and vector B.

In this example $\Sigma_i a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3$

$A_1 = 1$
$A_2 = 2$
$A_3 = 3$
$B_1 = -1$
$B_2 = 2$
$B_3 = -3$

Plug into the formula:

$\Sigma_i a_i b_i = 1(-1) + 2(2) + 3(-3) = -1 + 4 + -9 = -10 + 4 = -6$
$\Sigma_i a_i b_i = -6$

Now we need to find out what |A| |B| is,

Well luckily this is really easy, it's just pythagorean theorem, but in three dimensions.

Recall that in two dimensions pythag states that

$c^2 = a^2 + b^2$

In three dimensions, pythag states that

$d^2 = a^2 + b^2 + c^2$

C is the value of |A| when A is a vector defined in two dimensions, likewise, d is the value of |A| when A is a vector defined in three dimensions.

Fun fact time! This trend continues for any n number of dimensions!

Therefore to extend this concept to the problem we are tasked with solving,

$$|A| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

In the same way,

$$|B| = \sqrt{b_x^2 + b_y^2 + b_z^2}$$

Plugging into the above equations,

$$|A| = \sqrt{(1)^2 + (2)^2 + (3)^2}$$

$$|B| = \sqrt{(-1)^2 + (2)^2 + (-3)^2}$$

Therefore,

$|A| = \sqrt{14}$

$|B| = \sqrt{14}$

Therefore, $|A|\,|B| = 14$

Plugging into the angle equation, we get that

$\theta = \cos^{-1}(-6/14) = \cos^{-1}(-3/7)$

Cross product:

The Cross product operator is a vector operator that determines the degree of "dislikeness", basically it's the exact opposite of the dot product in every aspect. Meaning this is a lot harder to compute unfortunately :(

Important things to note about the cross product:

1. The result of the cross product operator is a ***VECTOR*** quantity, meaning it has a magnitude ***AND*** a direction.
2. To determine the direction of the cross product, you must use ***the right hand rule*** for cross products.
3. If the cross product of two vectors evaluates to 0, those vectors are said to be parallel, or antiparallel. We will discuss the rationale behind this statement later on when we learn how to compute cross products of vectors.
4. The cross product gives us an indication of the area of the parallelogram formed by the vectors.
5. The direction of the cross product vector is always perpendicular to the two vectors.
6. The cross product is NOT Commutative, A x B =/= B x A, but A x B = -1(A x B)

How to compute the cross product of two vectors, A and B.

There are two ways of computing the cross product or vector product, one is very easy and the other is much harder, but doesn't require knowledge of the angle between the vectors.

Equation 1:

$A \times B = |A|\,|B| \sin\theta$

Because sin(0) = 0 and sin(π) = 0, if the cross product is 0, the vectors are said to be parallel or antiparallel respectively.

Equation 2:

A x B = determinant of the following matrix

i	j	k
a_i	a_j	a_k
b_i	b_j	b_k

$A \times B = \widehat{i}\,(a_j b_k - a_k b_j) - \widehat{j}\,(a_i b_k - b_k b_i) + \widehat{k}\,(a_i b_j - a_j b_i)$

Easy way of memorizing this formula because i know it's very daunting when you first see it, cross out the row below and across from the unit vector you want and then there will be a smaller, 2 x 2 matrix that you can look at. Do top left times bottom right minus top right times bottom left, it'll make a cross/x pattern. Rinse and repeat for all the unit vectors. Only caveat with this technique is that you need to remember the minus sign before the j unit vector.

What you're really doing is finding what is called the "determinant" of the original matrix and then evaluating the mini 2 x 2 matrices.

Uses of the cross product in a realistic setting:

1. Find orthogonal vector
2. See if two vectors are parallel
3. Find the angle between two vectors

Given the fact that $A \times B = |A|\,|B| \sin\theta = \widehat{i}\,(a_j b_k - a_k b_j) - \widehat{j}\,(a_i b_k - b_k b_i) + \widehat{k}\,(a_i b_j - a_j b_i)$

$\sin\theta = [\widehat{i}\,(a_j b_k - a_k b_j) - \widehat{j}\,(a_i b_k - b_k b_i) + \widehat{k}\,(a_i b_j - a_j b_i)] / |A|\,|B|$

$\theta = \sin^{-1}([\widehat{i}\,(a_j b_k - a_k b_j) - \widehat{j}\,(a_i b_k - b_k b_i) + \widehat{k}\,(a_i b_j - a_j b_i)] / |A|\,|B|)$

Because this is a really disgusting process and it's extremely labor intensive, if you want to check to see if you did the cross product correctly, you can check it using the dot product.

Because the cross product always makes a vector, C that is orthogonal to the parent vectors, A and B, C dot A must equal 0, and C dot B must also equal 0. This is because C dot A = |C| |A| cosθ and cos(90) = 0.

Unit 2: Calculus

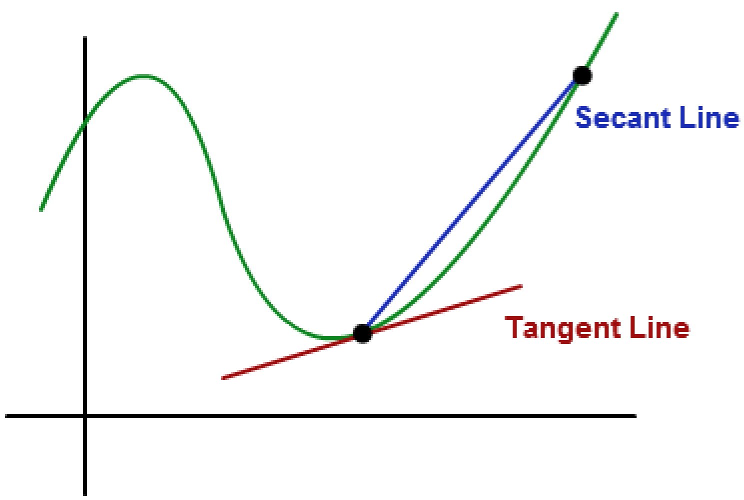
$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

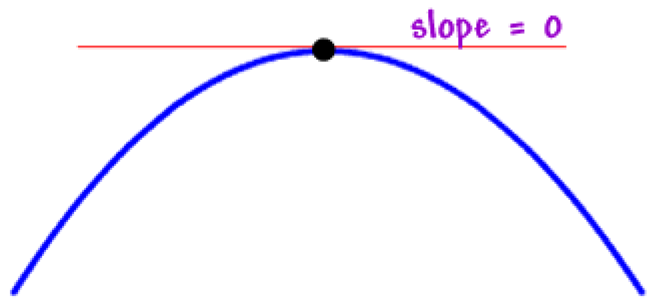
KEEP
CALM
AND
DO MORE
CALCULUS

Derivatives:

What is a derivative?

Well the derivative is a function that models the slope of the tangent line on a certain point of the parent function. It's basically a way to find the slope between two infinitely close points.





Let's suppose we have a differentiable function (a function for which the derivative exists) called f(x), contained on this function are the points (x, f(x)) and (x+h, f(x+h)).

Recall difference quotient,

Average Rate of Change = $\frac{f(x+h) - f(x)}{x+h - x} = \frac{f(x+h) - f(x)}{h}$

Well if the points are infinitely close together, the value of h will approach 0, because at the value of 0, the two points converge.

Therefore we get what mathematicians call the "limit definition of the derivative"

Instantaneous Rate of Change = $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Important thing to note is that the derivative itself is a function and what it does is tells us the instantaneous rate of change of a function at a given point.

To denote the derivative of f(x) mathematicians use the notation f'(x) read as f prime of x. Or, if you're a purist you can write it as $\frac{d f(x)}{dx}$ which is read as "the derivative of f(x) with respect to x"

Example)

$f(x) = x^2$
$f'(x) = ?$

To find the derivative of f(x) we need to use the limit definition of the derivative since that's the only method we know of right now.

Recall the limit definition of derivative:

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

If we plug in h is 0 into the above equation, we get 0/0 which is no bueno, so we need to do some algebraic manipulation.

$f(x) = x^2$
$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$

Plugging that into the limit definition of derivative yields the following:

$$\lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = f'(x)$$

That looks really messy, but it actually is quite nice because things cancel.

$$\lim_{h\to 0} \frac{2xh + h^2}{h} = f'(x)$$

If we plug in h = 0 into the above equation we still get 0/0, so we need to continue to simplify.

$$\lim_{h \to 0} \frac{h(2x+h)}{h} = f'(x)$$

The h's cancel leaving us with the following equation:

$$\lim_{h \to 0} 2x + h = f'(x)$$

Plugging in h = 0 gives us

f'(x) = 2x + 0

f'(x) = 2x

Notice anything special about the exponent of the parent function and the coefficient of the derivative function?

Power rule:

Power rule can only be used for polynomial functions, or functions that have a varying base, but constant exponent i.e. x^2

Power rule states that for any polynomial function expressed in terms of x^n its derivative is nx^{n-1}

Examples:

1) f(x) = x^3 f'(x) = $3x^2$
2) f(x) = x^4 $f'(x) = 4x^3$
3) f(x) = x $f'(x) = 1x^0 = 1$
4) f(x) = $2x^3 = 6x^2$

Constant rule:

Because the derivative represents the slope of the tangent line of the parent function at a given point, if the function does not increase or decrease ever, the derivative of it is 0. The only case that this occurs is when the function we are taking the derivative of is in the form f(x) = k where k is some constant such as 1,2,3,4, etc.

Ie. f(x) = 141341823471928043018347140 f'(x) = ?

Well f(x) doesn't vary in terms of x so when we take its derivative it just turns out to be 0, because that function is a horizontal line and so the slope of the tangent line is 0 always.

f'(x) = 0.

Trigonometric functions:

f(x) = sin(x)

f’(x) = cos(x)
f’’(x) = -sin(x)
f’’’(x) = -cos(x)
f’’’’(x) = sin(x)

Y = tan(x)
Y’ = $sec^2(x)$

g(x) = cot(x)
g’(x) = $-csc^2(x)$

h(x) = sec(x)
h’(x) = sec(x) tan(x)

j(x) = csc(x)
j’(x) = -csc(x) cot(x)

Chain Rule:

If a function is written in terms of a composition of functions then you must use chain rule.

If y is a composition of f(x) and g(x), y’ = f’(g(x)) g’(x)

Example)

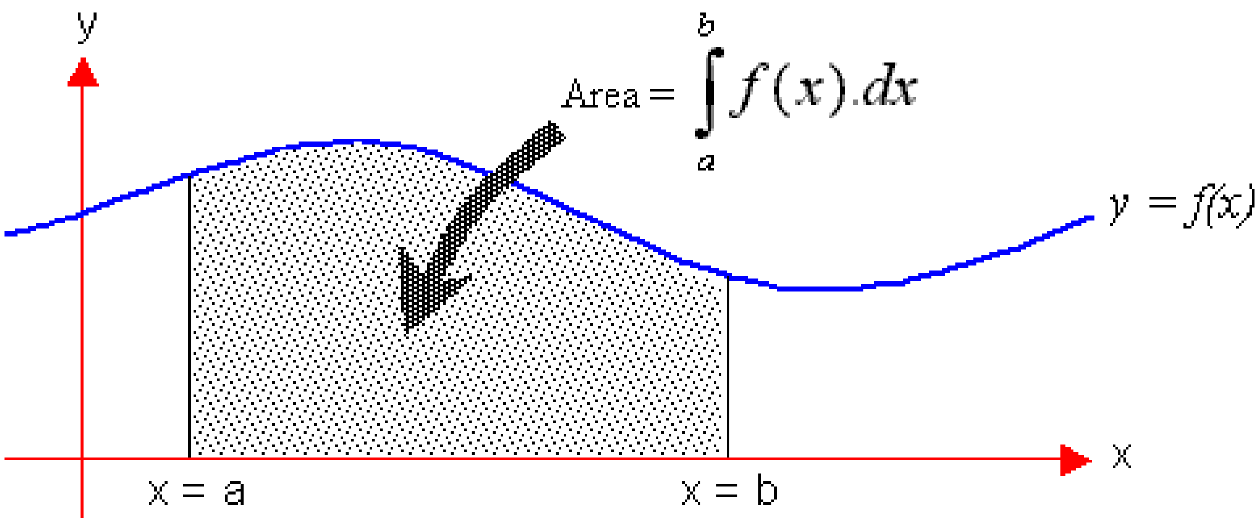
$Y = sin(x^2)$
$Y' = cos(x^2)\ 2x$

$f'(g(x)) = \cos(x^2)$
$g'(x) = 2x$
$f(x) = \sin(x)$
$g(x) = x^2$

Example)

$Y = tan^3(2x^2)$
$Y' = 3tan^2(2x^2)\ 4x\ sec^2(2x^2) = 12xtan^2(2x^2)sec^2(2x^2)$

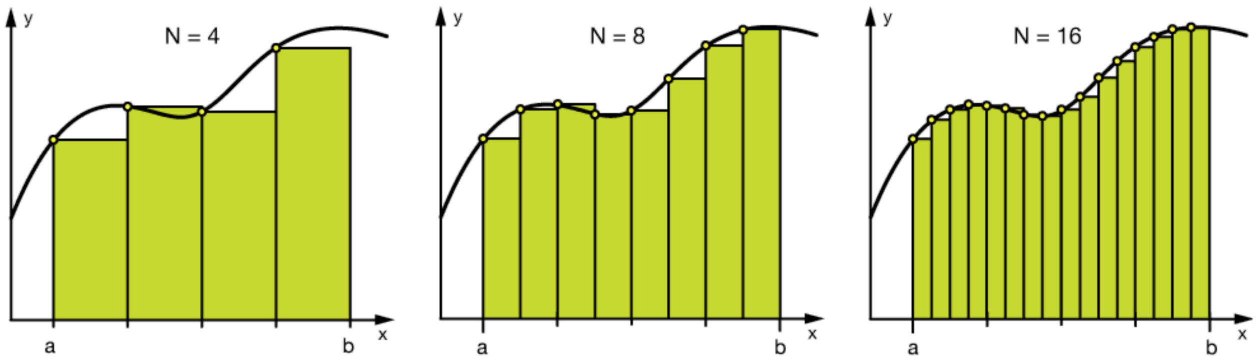
Integrals:



What is an integral?

Well in its simplest form, the integral is a way of calculating area under a curve, much of the time, calculating the area between a given curve and the x-axis, though not always.

Way before any of us were born there was a man named Reimann and he became famous for what is called a Riemann Sum, which is the way that the old people did area under the curve calculations. Basically what he did was he made a bunch of rectangles underneath the curve, what he found was that the more rectangles you had underneath the curve, the more accurate the area measurement was.



As you can see, there is less white space in the last picture between x = a and x = b than there is in the other pictures, hence the area is more precise as the number of rectangles approaches ∞.

Recall that Area of a rectangle is A = base * height.

Well the height of the rectangle is dictated by the curve in question, or y, and the base of the rectangle is dictated by the number of rectangles in the range given.

Hence Riemann stated that

$$\text{Area} = \lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{b-a}{n}\right) f(x_k)$$

But that looks really disgusting so instead mathematicians write that same thing like this, in terms of an ***integral***.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{b-a}{n}\right)f(x_k)$$

The integral $\int_a^b f(x)\,dx$ is really saying, add up the small areas of the rectangles encapsulated by the curve f(x) on the interval [a,b] , where the height of the rectangles is f(x) and the base of the rectangles is dx, an infinitely small range of x values.

Well now that we know the history and what visually the integral is, let's focus ourselves on actually computing one!

Anti-power rule:

This is basically gonna be like power rule for derivatives, but it's going to be the exact opposite. Anti-power rule states for an integrable function, f(x) expressed in terms of a polynomial,

$\int f(x)\,dx \;=\; \int x^n\,dx \;=\; x^{n+1}\frac{1}{n+1} + C$ this only works if n $\neq$-1

Memorized ones:

$\int \frac{1}{x} dx = ln(x) + C$

Integrals are the opposite of derivatives, if you take the derivative of the integral of a function, you get whatever is in the integrand.

I.e. $\int 2x\, dx = x^2 + C$ = y

Y’ = 2x

Recall that y’ can be written as $\frac{dy}{dx}$

$\frac{dy}{dx}$ = 2x

$dy = 2x\, dx$

Integrate both sides

Remember integrals are really adding stuff up, so if you add up a bunch of small chunks of y, or dy, then you get the whole y.

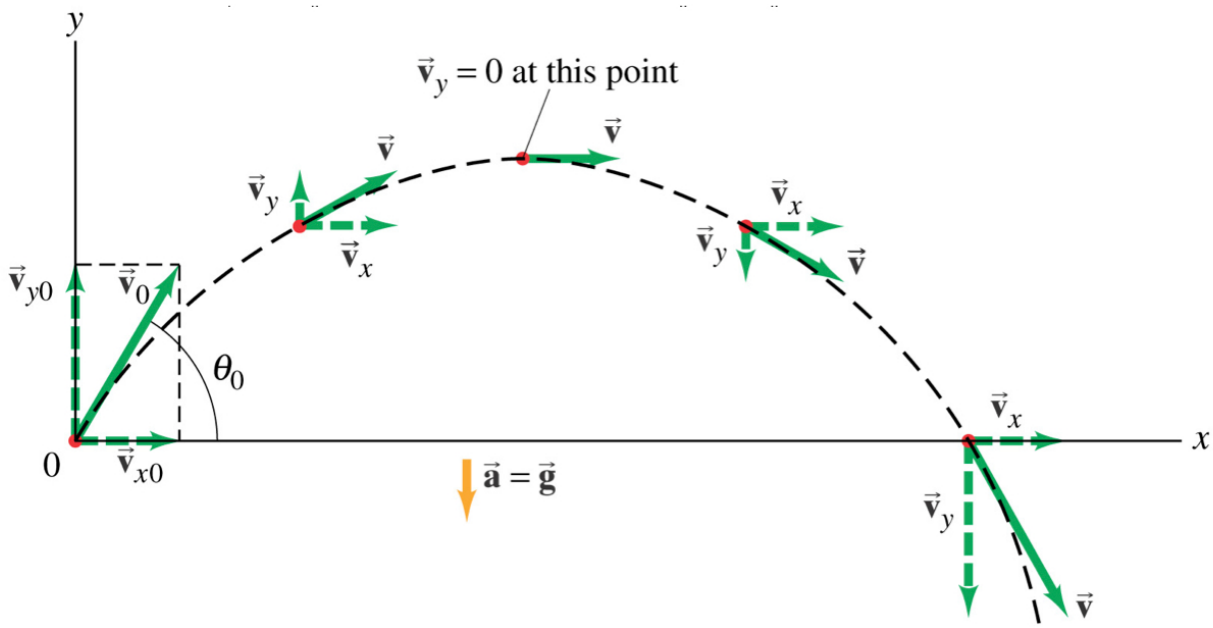
Therefore $\int dy = y$.

$$\int dy = \int 2x \, dx$$

$$y = x^2 + C$$

The C results because technically if we took the derivative of y with respect to x we would get 2x regardless of what # C is, be it 1,2,3, 248932843848, etc. so C is just an arbitrary constant that you can solve for if they give you a point that lies on the graph y.

Unit 3: Kinematics



$X_f = X_i + V_i t + \frac{1}{2} at^2$

$V_f = V_i + at$

$V_f^2 = V_i^2 + 2a\Delta x$

$V_{avg} = \frac{\Delta x}{\Delta t}$

$V = \frac{dx}{dt}$

$A_{avg} = \frac{\Delta v}{\Delta t}$

$A = \frac{dv}{dt}$

$A = \frac{d^2x}{dt^2}$

$V(t) = \int a(t)\, dt$

$X(t) = \int v(t)\, dt$

Perpendicular vectors are independent of each other.
Rest means V = 0 m/s

Two dimensional kinematics:

Procedure:
Separate the x and y components of the motion.
REMEMBER VELOCITY, ACCELERATION, AND DISPLACEMENT ARE ALL VECTOR QUANTITIES!!!! THEY HAVE DIRECTION SO PICK A POSITIVE AND NEGATIVE DIRECTION.

Example:

A soccer player kicks a soccer ball with an initial velocity of 50 m/s at an angle of 30 degrees with respect to the horizontal.

X direction	Y direction
V_i = 50 m/s * cos(30)	V_i = 50 m/s * sin(30)
V_f = Vi	V_f = n/a
A = 0 m/s^2	a = -g

At time t/2 the velocity of the soccer ball in the y direction is 0 m/s, remember SYMMETRY IS YOUR FRIEND!

When a is 0 the only formula you can use that will help you is

$$X_f = X_i + V_i t + \frac{1}{2} at^2$$

Most often we make the initial position of the object = 0 therefore the formula is

$$X_f = V_i t + \frac{1}{2} at^2$$

In the x direction a is 0 therefore

$$X_f = V_i t$$

Example:

A rock rolls down a 50 m cliff with a horizontal speed of 10 m/s, where should Joe put his target to have the rock hit it exactly on the bullseye?

Right is positive, down is positive

X direction | Y direction

X = ?	X = 50 m
V_i = 10 m/s	v_i = 0 m/s
V_f = 10 m/s	v_f =
A =0 m/s^2	a = g
T =	t = ?

REMEMBER THE TIME IN BOTH DIMENSION IS THE SAME!!!

Y direction:

$X_f = X_i + V_i t + \frac{1}{2} at^2$

$X_f = \frac{1}{2} at^2$

$2X_f = at^2$

$2X_f/a = t^2$

$T = \sqrt{2X_f/a}$

$T = \sqrt{2X_f/g}$

$T = \sqrt{2(50m)/10m/s^2}$

$T = \sqrt{10}$ s

X direction:

$X_f = X_i + V_i t + \frac{1}{2} at^2$

$X_f = V_i t$

$X_f = 10 \text{ m/s} * \sqrt{10} \text{ s}$

Unit 4: Dynamics

Newton's three laws:

1. A body in motion tends to stay in motion, also called the law of inertia.
2. Our best friend aka $F_{net} = ma$
3. Every action has an equal and opposite reaction.

Inertia:
Inertia α mass.
Resistance to change in velocity.

F_{net} = ma:
Remember a force is a vector quantity and it has direction. Make sure your positive and negative directions are consistent. IF THE VELOCITY IS CONSTANT, THE OBJECT IS IN EQUILIBRIUM!

$F_{net} = m \frac{dv}{dt}$

$F_{net} = m \frac{d^2x}{dt^2}$

$F_{fs} \le \mu_s F_n$

$F_{fk} = \mu_k F_n$

$F_{drag} = kv$ or kv^2

$F_{spring} = -kx$

$F_{gravity} = mg$

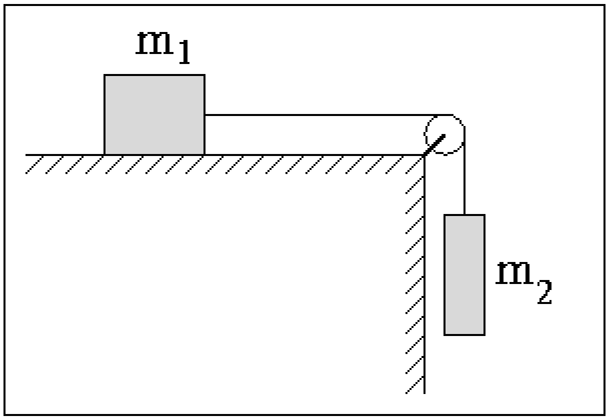
$F_{gravity} = G \frac{m_1 m_2}{r^2}$

$F_{electrostatic} = qE$

$F_{electrostatic} = k \frac{q_1 q_2}{r^2}$

$F_{centripetal} = m \frac{v^2}{r}$

Example: masses m_1 and m_2 are attached to a massless rope and



a pulley of negligible mass and frictionless girders. m_1 is situated on a frictionless surface, and when the masses are released, m_2 descends with an acceleration, a. Find the tension in the rope.

Setup:

Draw free body diagrams of each mass individually
Down is positive and right is positive

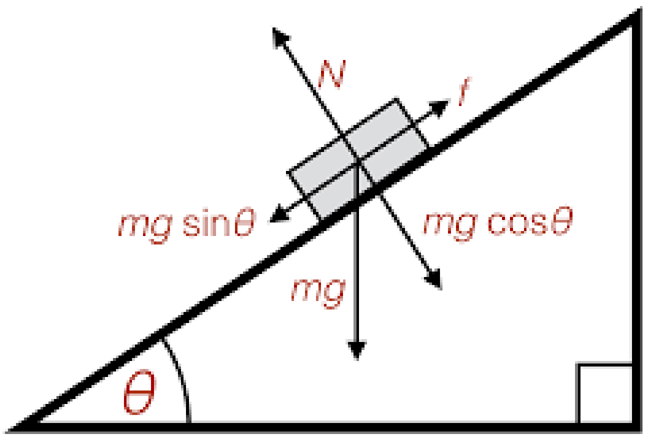
FBD of m_2
$F_{tension}$ upwards, $F_{gravity}$ downwards

$\Sigma F_y = F_{gravity} - F_{tension} = m_2 a$
$\Sigma F_y = F_{gravity} - m_2 a = F_{tension}$
$\Sigma F_y = m_2 g - m_2 a = F_{tension}$

Therefore the tension is $m_2g - m_2a$

Example:

An object of mass m is on an inclined plane and is sliding down



at a constant velocity as shown in the diagram. Find the coefficient of kinetic friction for which the box would have a constant velocity.

Down the ramp is positive, into the ramp is positive.

$\Sigma F_x = ma_x$

$\Sigma F_x = mg\sin\theta - F_{fk} = 0$

VELOCITY IS CONSTANT THEREFORE IT IS IN EQUILIBRIUM!!!

$\Sigma Fx = mg\sin\theta = F_{fk} = \mu_k F_n$

$\Sigma F_y = mg\cos\theta - F_n = ma_y$

$\Sigma F_y = mg\cos\theta = F_n$

$\Sigma F_x = mg\sin\theta = \mu_k F_n$
$\Sigma F_x = mg\sin\theta = \mu_k mg\cos\theta$

Therefore,

$\mu_k = mg\sin\theta / mg\cos\theta$
$\mu_k = \sin\theta / \cos\theta$
$\mu_k = \tan\theta$

Remember that $F_{net} = ma$ can be used to bridge kinematics with the dynamics that we learned in this chapter.

For uniformly accelerated motion that is caused by a force applied, the acceleration is such that

$F_{net} = ma$
Therefore
$A = \frac{F_{net}}{m}$

Uniform circular motion:

$F_{net} = ma_c$
$A_c = \frac{v_t^2}{r}$
$T = \frac{1}{f}$
$f = \frac{1}{T}$

Recall $v_t = r\,\omega$

Therefore,

$$A_c = \frac{(r\omega)^2}{r}$$

$$A_c = \frac{r^2\omega^2}{r}$$

$$A_c = r\omega^2$$

The object is moving in a circle, therefore it is moving a distance 2πr (circumference of the circle) in a time T (the period of the motion)

Therefore,

$$V_t = \frac{2\pi r}{T}$$

Therefore,

$$A_c = \frac{(\frac{2\pi r}{T})^2}{r}$$

$$A_c = \frac{4\pi^2 r^2}{T^2 r}$$

$$A_c = \frac{4\pi^2 r}{T^2}$$

$$A_c = 4\pi^2 r f^2$$

In any circular motion problem, YOU NEED TO IDENTIFY WHICH FORCE IS CAUSING IT TO GO IN A CIRCLE!!!

Example) A planet, P with mass m_p is revolving around a star, S with an orbital velocity of v and mass m_s. Find the distance the planet is away from the sun.

$\Sigma F_c = ma_c$

$F_{gravity} = m_p \frac{v_t^2}{r}$

$Gm_pm_s/r^2 = m_p \frac{v_t^2}{r}$

$Gm_pm_s/r = m_pv^2$

$Gm_s/v^2 = r$

Find the period of the motion.

$\Sigma F_c = ma_c$

$Gm_pm_s/r^2 = m_p \frac{v_t^2}{r}$

$Gm_s/r^2 = \frac{v_t^2}{r}$

$Gm_s/r = v^2$

$Gm_s/r = (\frac{2\pi r}{T})^2$

$Gm_s/r = \frac{4\pi^2 r^2}{T^2}$

$\frac{Gm_s}{4\pi^2 r^3} = \frac{1}{T^2}$

$\frac{4\pi^2 r^3}{Gm_s} = T^2$

$T = \sqrt{\frac{4\pi^2 r^3}{Gm_s}}$

Plugging in the formula for the distance between the planet and the star

$$T = \sqrt{\frac{4\pi^2 (Gm_s/v^2)^3}{Gm_s}}$$

Nonuniform circular motion:

Sometimes, such as the case in a vertical loop, the force causing the centripetal acceleration is not constant.

The force that points in the direction of the center of the circle is gravity when the object is on the top of the loop and is barely holding on.

Therefore,

$\Sigma F_c = ma_c$

$F_{gravity} = ma_c$

$Mg = ma_c$

$g = \frac{v^2}{r}$

$gr = v^2$

$V = \sqrt{rg}$

At the bottom of the ramp, the forces acting on it is the normal force and the force due to gravity.

$\Sigma F_c = ma_c$

$F_n - F_{gravity} = ma_c$

$F_n = ma_c + F_{gravity}$

Therefore, the normal force needs to be greatest at the bottom of the ramp and so if you were to swing an object on a string and made a vertical circle with it, the object would most likely break at the bottom of the loop.

It is important to note that the normal force acting on the top of the ramp as opposed to the bottom of the ramp differs by a factor of 2mg.

Top of the ramp:

$\Sigma F_c = ma_c$

$F_n + F_{gravity} = ma_c$

$F_n = Ma_c - F_{gravity}$

Bottom of the ramp:

$\Sigma F_c = ma_c$

$F_n - F_{gravity} = ma_c$

$F_n = Ma_c + F_{gravity}$

Unit 5: Work, power, and energy

Work done by a Force:

In General:

$$W = \int_a^b F \star dr$$

REMEMBER:

Work can be a <u>negative</u> measurement and it is the dot product of the force vector and the displacement vector.

If Force is constant then get the component that is causing it to bring in motion. This means breaking up it into X and Y components, or parallel and perpendicular components

$$W = Fx\cos\theta$$

$$W = F \bullet x$$

$$W_{total} = \Delta KE = KE_f - KE_i$$

Conservative Forces (Force due to gravity, etc.) does not depend on the path taken. Measure the start to end path and measure the work done

Non-Conservative forces (Force of friction, etc.) depend on the path taken. Measure the total Distance (x) and calculate the work done

Power:

Power is how fast work is done

$$P = Fv$$

$$P = \frac{dW}{dt}$$

Conservation of Energy:

$$KE_i + PE_{si} + U_i + W_{nc} = KE_f + PE_{sf} + U_f$$

Very rarely will all components be used. Remember to get each of the components of each force to get the work done on the object.

KE = $\frac{1}{2}mv^2$

$KE_{rotational}$ = ½ $I\omega^2$

U_g = $\frac{-Gm_1m_2}{r}$

(for things really far away like in space)

U_g = mgh
(for things close to the surface of the earth, this is because g varies as a function of distance, and close to the earth g is relatively constant.)

$W_{Fgravity} = -\Delta U_g$
$F_{gravity} = \frac{-dU_g}{dx}$
$F_{gravity}\, dx = -dU_g$

$\int F_{gravity}\, dx = -U_g$

$-\int F_{gravity}\, dx = U_g$

$U_{spring} = \frac{1}{2}kx^2$
(only for a spring that obeys Hooke's law)

Derivation of the U_{spring}

$F_{spring} = -kx$

Recall that in general $U_{conservative} = -\int F_{conservative}(x)\,dx$

Therefore,

$$U_{spring} = \int kx\,dx$$

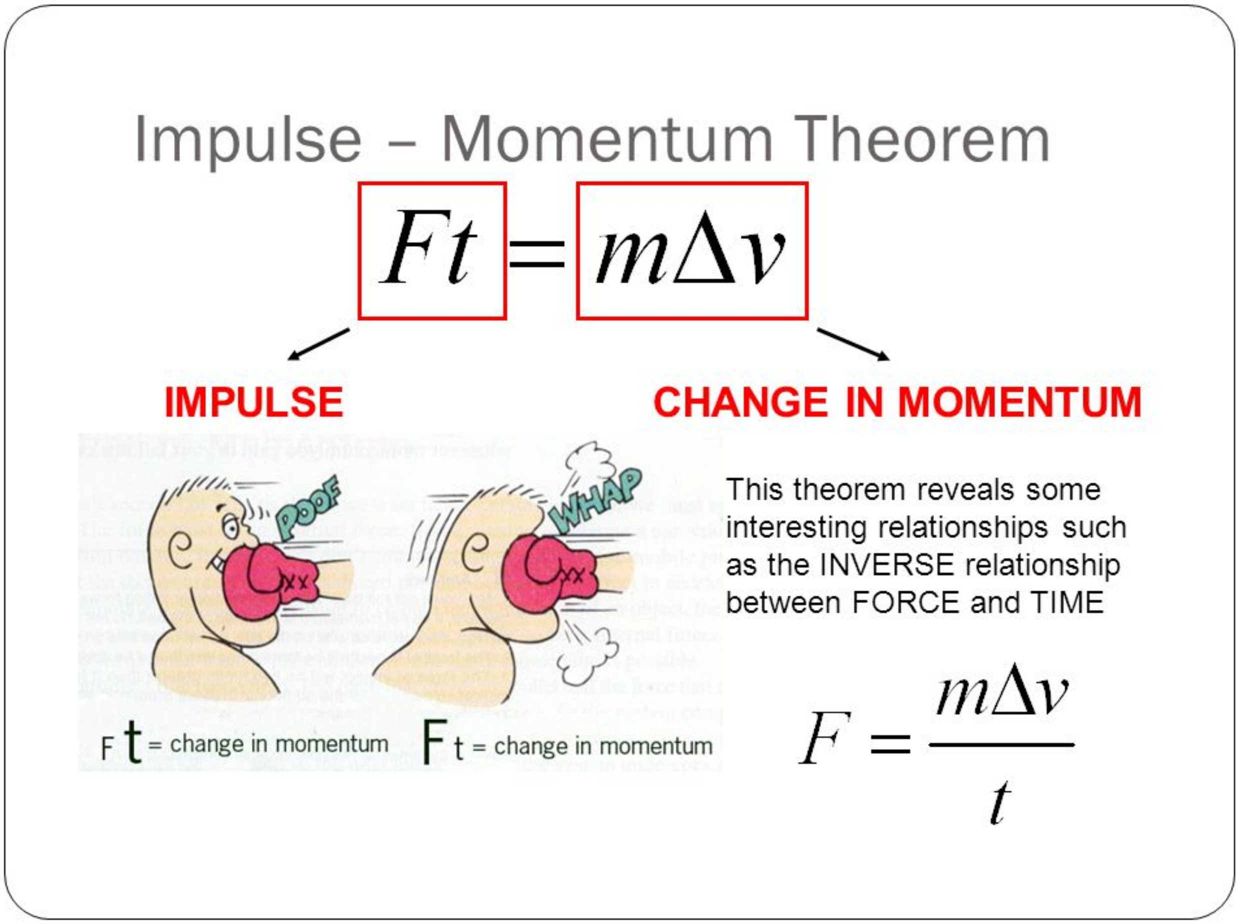
$U_{spring} = \frac{1}{2}kx^2$

$W_{nonconservative} = \Delta E$

Remember, Work is a vector dot product, so you need to find the component of the force that is in the direction of the displacement.

$W = F_x * x_x + F_y * x_y + F_z * x_z$

Unit 6: Center of mass and momentum



$F = ma$

$F = m\frac{\Delta v}{\Delta t}$

$P = mv$

$F = \frac{\Delta p}{\Delta t}$

$F\Delta t = \Delta p$

$\Delta p = J$

$J = mv_f - mv_i$

$\Sigma m * x_{cm} = \Sigma x*m$

$\int dm \ x_{cm} = \int x \, dm$

REMEMBER, LINEAR MOMENTUM IS ALWAYS CONSERVED IF THERE IS NO IMPULSE ACTING ON THE SYSTEM!!! THIS MEANS THAT IN ANY COLLISION, MOMENTUM IS ALWAYS CONSERVED!!

What is a system, though? Easiest way to explain a system is that if you were to draw an imaginary dotted line around the objects in question and no one is pushing or pulling from ***outside*** the dotted line, there is no impulse on the system, therefore, momentum is conserved.

Because velocity is a vector quantity, ***momentum is a vector quantity*** as well. Therefore, it has ***direction and a magnitude. Choose the direction*** you want to make positive and ***keep it consistent***. ***You must separate momentum into x,y, and z components if necessary*** and you are dealing with a 2 or 3 dimensional collision.

Types of collisions:

Elastic Collisions:

In an elastic collision, both momentum and total kinetic energy are conserved. Often times, this means that you will need to set up a system of equations to find the resultant velocities because this can occur in a number of ways. $KE_{final} = KE_{initial}$ and $P_{before} = P_{after}$. These almost never occur, the closest approximation we have to an elastic collision is collisions between gas molecules and even those aren't perfectly elastic, Cassin would be so proud :) .

Inelastic collisions:

Two cases for inelastic collisions, can either be inelastic or perfectly inelastic. Perfectly inelastic collisions are when the two objects stick together after the collision. In these types of collisions, KE is not conserved and some of it is lost as heat or another form of energy such as sound.

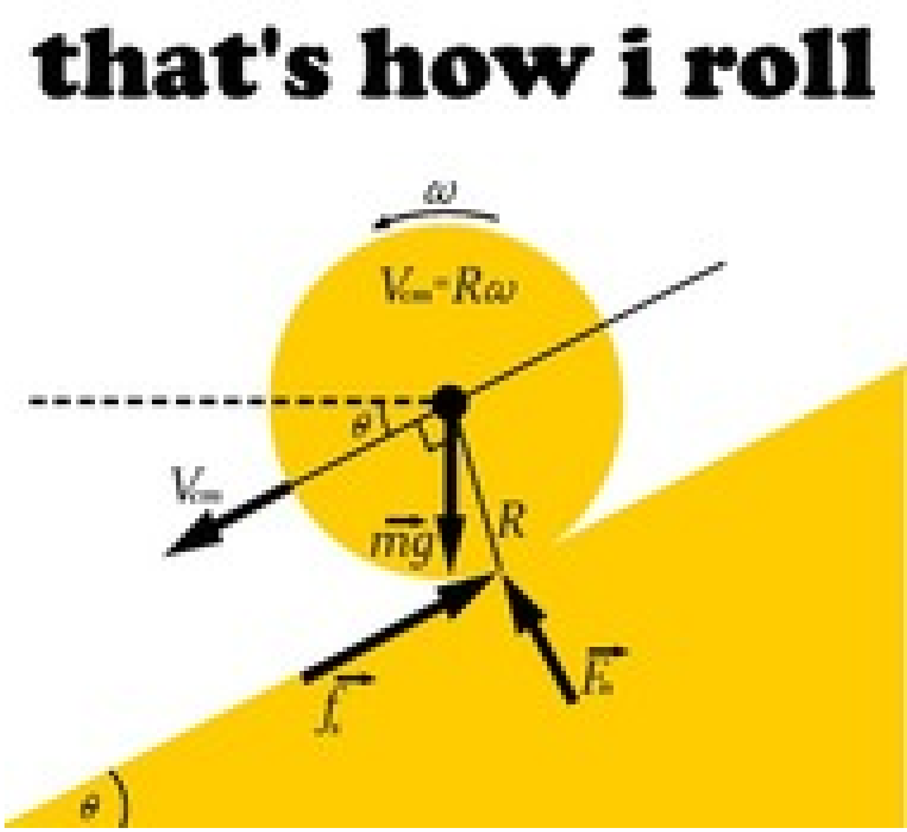
Example) on a highway intersection, Joseph is driving his Bugatti Veyron Supersport (mass 100 kg) and reaches a speed of 100 m/s (Joe is a bad boy and likes to live life on the edge) when he then collides with Ethan, who is driving his Lamborghini Aventador (mass 110 kg) at a speed of 85 m/s (Ethan is a bit more prudent) in the opposite direction. After the collision, the two cars stick together to form a composite body (poor Joe and Ethan), find the speed of this composite body.

$\Sigma P_{before} = \Sigma P_{after}$

$M_1 v_1 - m_2 v_2 = (m_1 + m_2)\ v_{composite}$

$V_{composite} = (M_1 v_1 - m_2 v_2)/(m_1 + m_2)$
Plug in and solve

Unit 7: Rotational motion



$S = \theta r$

$\omega_{average} = \frac{\Delta\theta}{\Delta t}$

$\omega = \frac{d\theta}{dt}$

$V_{center\ of\ mass} = \omega r$

$A_{center\ of\ mass} = \alpha r$

$\alpha_{average} = \frac{\Delta\omega}{\Delta t}$

$\alpha = \frac{d\omega}{dt}$

$\tau = r \text{ X } f = -1(F \text{ x } r)$

$\Sigma\tau = I\alpha$
*$I = mr^2$ only for a point particle, NOT a solid object.

**$I = \int r^2\, dm$

This equation is derived from the point particle equation for moment of inertia, basically we are isolating little point masses along the object which all have a mass, dm and we are adding all of their individual contributions to the total moment of inertia of the whole solid object. The whole is equal to the sum of its parts, therefore the integral is a valid equation.

I for a solid cylinder about center = $1/2\ MR^2$
I for a solid rod about center = $1/12\ MR^2$
I for solid rod about end = $1/3\ MR^2$
Parallel axis theorem: $I_{new} = I_{center\ of\ mass} + Md^2$

TORQUE IS A VECTOR QUANTITY AND IS A VECTOR PRODUCT OF DISPLACEMENT AND FORCE.

$T = Fx\sin\theta$

The direction of the torque is determined by the right hand rule, whereby you stick your fingers in the direction of the displacement and curl them toward the line of action of the force.

$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega_f^{\ 2} = \omega_i^{\ 2} + 2\alpha t$

$L = I\omega$

Angular momentum is always conserved so long as the external torque acting on the system is 0

$\Sigma\tau = I\,\alpha$

$\Sigma\tau = I\,\frac{d\omega}{dt}$

$\Sigma\tau = I\,\frac{d^2\theta}{dt^2}$

$\Sigma\tau = \frac{dL}{dt}$

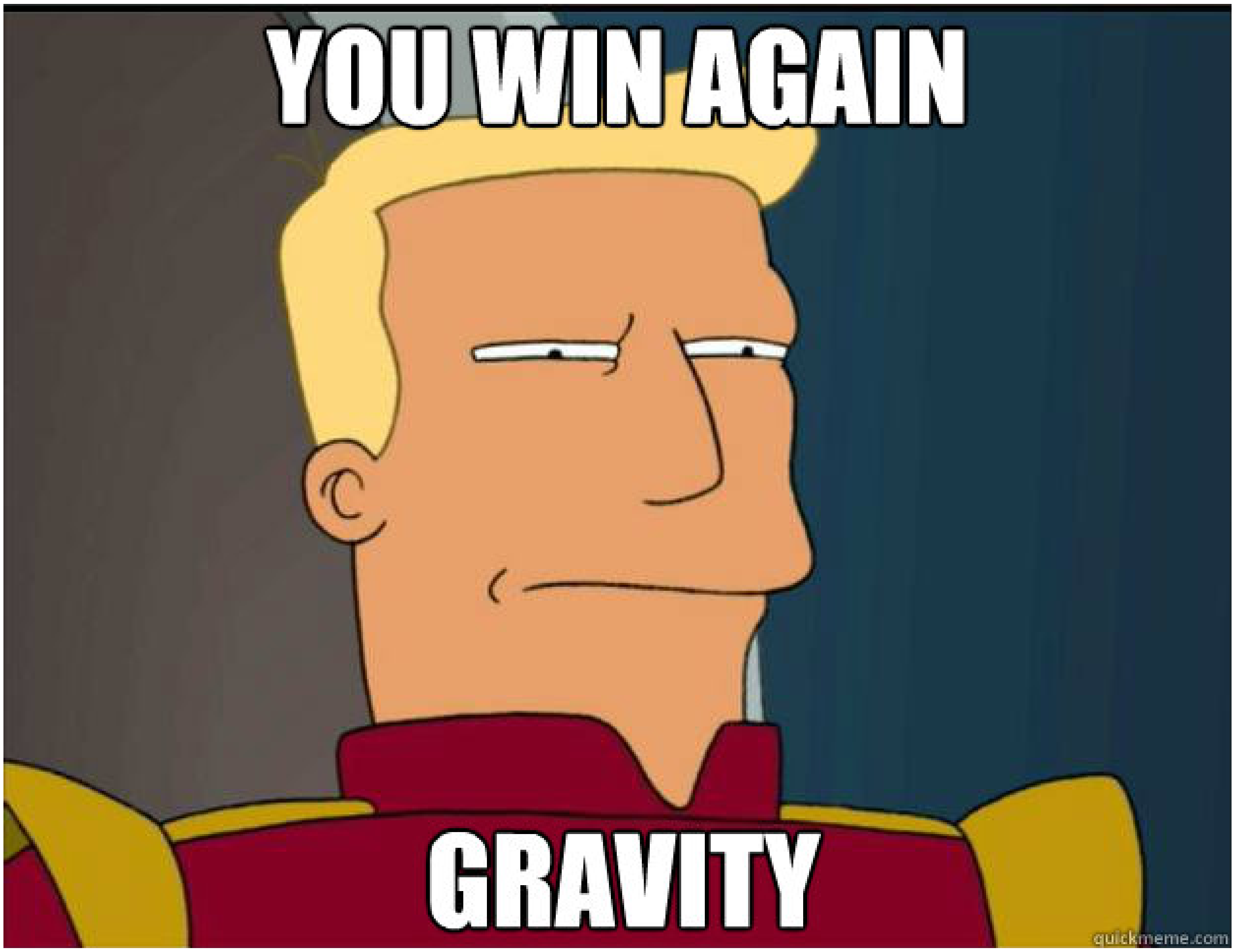
$KE_{rotational} = \frac{1}{2}I\omega^2$

Rolling without slipping only conditions:

$V_{cm} = v_t = r\omega$

$A_{cm} = a_t = r\alpha$

Unit 8: Simple Harmonic Motion and Gravity



$X(t) = A\cos(\omega t)$

$V(t) = \frac{d\,x(t)}{dt}$

$V(t) = -A\omega\sin(\omega t)$

$A(t) = \frac{d\,v(t)}{dt}$

$A(t) = -A\omega^2\cos(\omega t)$

$T = 2\pi \frac{1}{\omega}$

$V_{max} = A\ \omega$

$a = -\omega^2 x$

ω angular speed = ω angular frequency

$f = 1/T$

$T = 1/f$

T for a mass spring system only = $2\pi \sqrt{\frac{m}{k}}$

T for a pendulum only = $2\pi \sqrt{\frac{l}{g}}$

T for a solid pendulum = $2\pi \sqrt{\frac{2l}{3g}}$

$F_{gravity} = mg = G \frac{m_1 m_2}{r^2}$ = (m $\frac{v^2}{r}$ ONLY IF ITS IN ORBIT AND THE GRAVITATIONAL FORCE ACTS AS THE CENTRIPETAL FORCE)

$$\Phi_{gravitational} = \oint g \bullet dA = -G4\pi M_{enclosed}$$

$$\Phi_{gravitational} = gA$$

$$g = \frac{F_{graviational}}{m}$$

$$g = G \frac{m_1}{r^2}$$

Orbital velocity determined by the following setup:

$\Sigma F_c = ma_c$

$F_{gravity} = m \frac{v^2}{r}$

$G \frac{m_1 m_2}{r^2} = m_2 \frac{v^2}{r}$

Solving for V:

$G \frac{m_1}{r} = v^2$

Therefore V = $\sqrt{G\frac{m_1}{r}}$

Escape velocity is determined by the following setup:

$ME_f = ME_i + W_{nonconservative}$

$W_{nonconservative} = 0$,Therefore, $ME_f = ME_i$

$$\frac{1}{2}mv_{escape}{}^2 + \frac{-Gm_1m_2}{r_p} = \frac{1}{2}mv_f{}^2 + \frac{-Gm_1m_2}{r}$$

Escape velocity is defined as the minimum speed that you must attain in order to escape the gravitational pull of earth or some planet therefore $v_f = 0$

Because the object approaches an infinite distance away from the planet, to calculate the gravitational potential energy at that distance, we must take $\lim_{r\to\infty} \frac{-Gm_1m_2}{r}$ when we evaluate this limit, we get $\frac{-Gm_1m_2}{\infty}$

which gives us $\lim_{r\to\infty} \frac{-Gm_1m_2}{r} = 0$

This makes the right side evaluate to 0

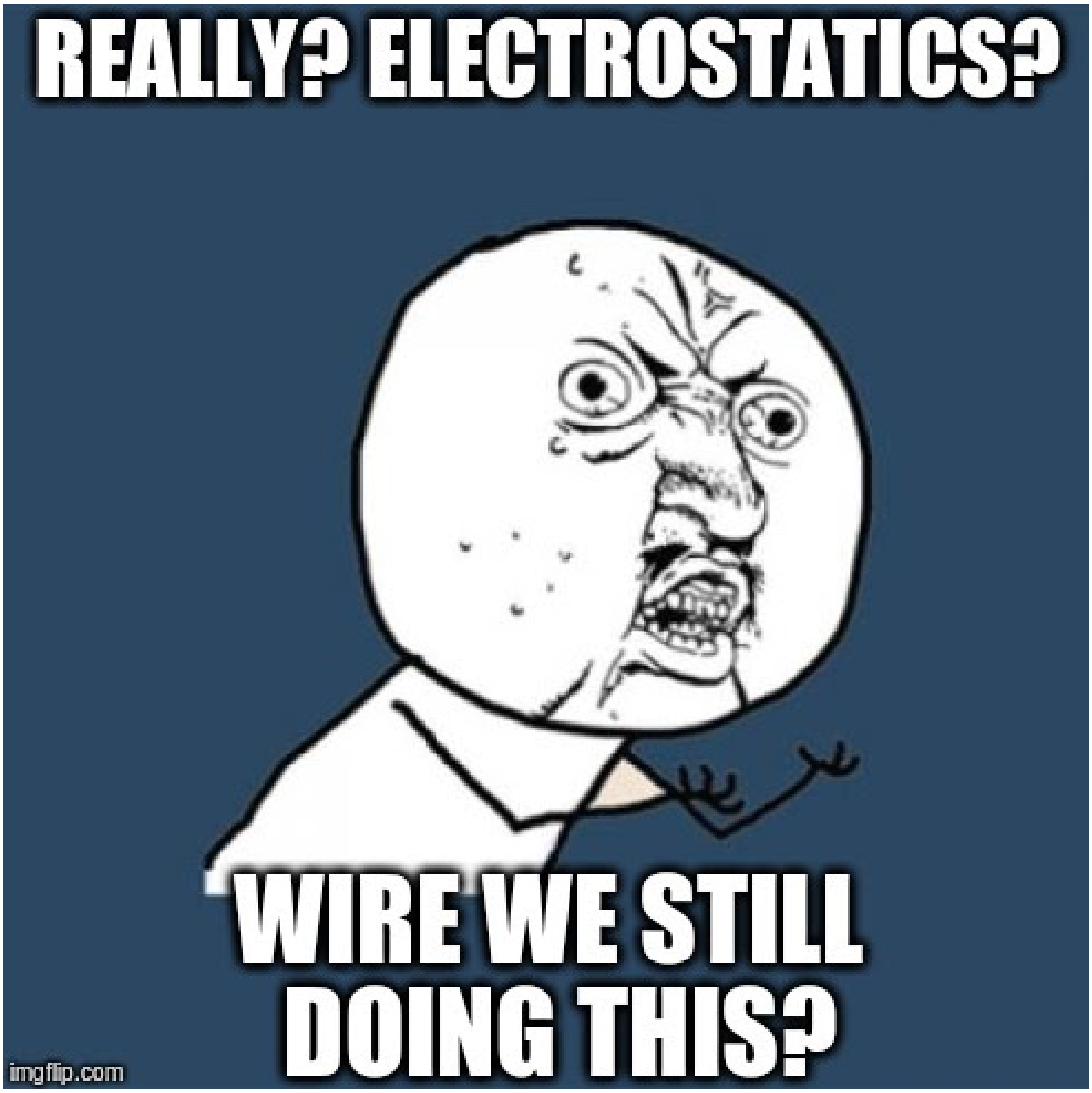
Therefore: $\frac{1}{2}mv_{escape}{}^2 = \frac{Gm_1m_2}{r_p}$

Therefore: $v_{escape}{}^2 = 2\frac{Gm_1m_2}{r_p}$

Therefore: $v_{escape} = \sqrt{2\frac{Gm_1m_2}{r_p}}$

Note that the escape velocity differs from orbital velocity by a factor of $\sqrt{2}$

Unit 9: Electrostatics



$F_{electrostatic} = qE$

$F_{electrostatic} = k\ \frac{q_1 q_2}{r^2}$

$E = k\ \frac{q_1}{r^2}$

$E = \frac{F_{electrostatic}}{q}$

$$U_{electrostatic} = k\ \frac{q_1 q_2}{r}$$

$$U_{electrostatic} = qV$$

$$V = \frac{U_{electrostatic}}{q}$$

$$V = k\ \frac{q_1}{r}$$

$$V = \int k\ \frac{dq}{r}$$

$$E = \int k\ \frac{dq}{r^2}$$

$$V = E\ x$$

$$V = -\int_{\infty}^{p} E\ dx$$

$$E = \frac{-dV}{dx}$$

$$W_{electrostatic} = -\int_{\infty}^{p} F_{electrostatic}\ dx$$

$$\Phi_{electric} = E \bullet A$$

$$\Phi_{electric} = \oint E \bullet dA = \frac{q}{\epsilon_0}$$

$$\lambda = \frac{q}{X}$$

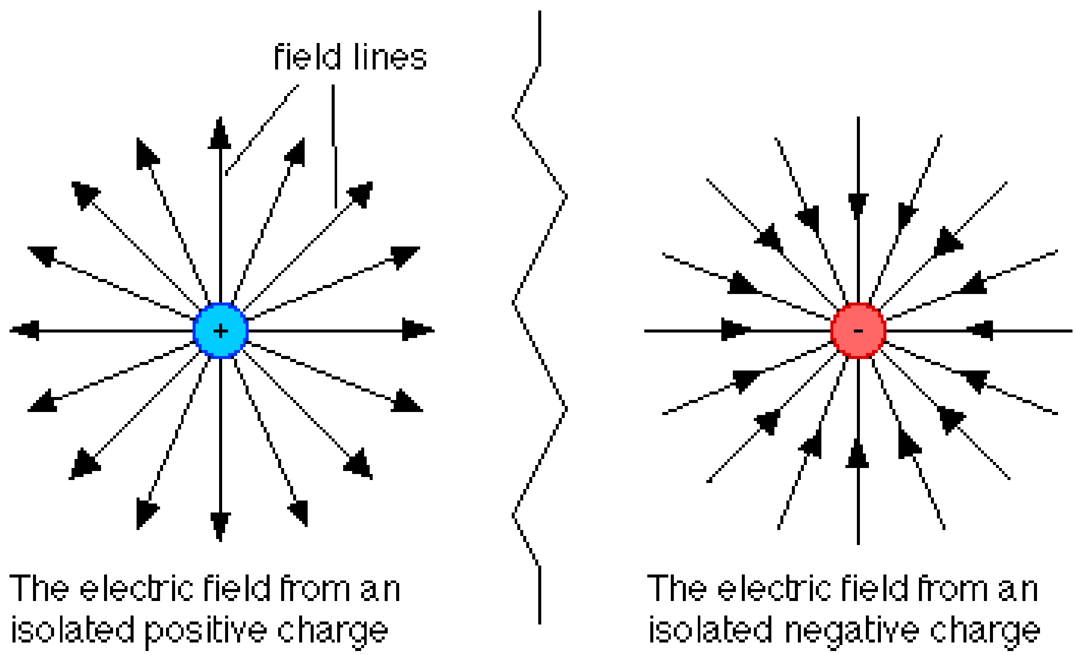
$$\sigma = \frac{q}{A}$$

$$\rho = \frac{q}{V}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

How to draw electric field lines:

In order to draw electric field lines, you must pretend to place a tiny imaginary positive charge, or TIPC and see how it would react to the source charge. Or in other words, if it would attract or repel.

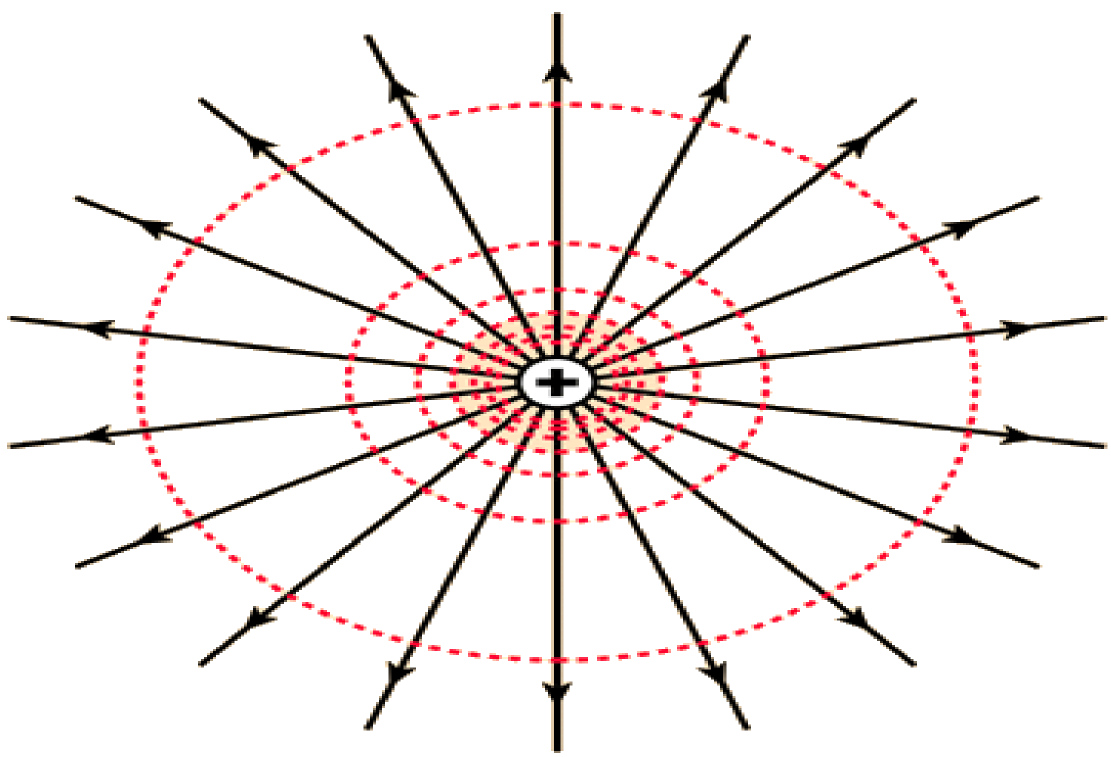


Because like charges repel, we see that the electric field lines are away from the positive charge, and because opposites attract, we see that the electric field lines are towards the negative charge.

Electric field lines CANNOT intersect and they must hit the source charge at a 90 degree angle.

Equipotential lines:

Lines that are drawn that indicate that the work done to move a positive charge to that location from infinitely far away is the same all along the line. Any movement along an equipotential line requires 0 J of work to be done.



See above figure, the red dotted lines are equipotential lines because from all directions it requires an equal amount of work to move a charge to any point along the dotted line.

What is Electric Potential Energy (EPE or $U_{electrostatic}$)?

Electric potential energy is the amount of energy stored in the electric field at a given location. In other words, how much energy must be exerted to move a test charge from an infinite distance away to the point of interest. The Electric Potential Energy can be computed easily using the work formula, see the work chapter to get brushed up on that. The negative sign in the formula included in the beginning of this chapter is included because the force applied to move the force is in the direction opposite that of the electrostatic force. The rest should be stuff that should be intuitive. The fact that the electrostatic force has a potential energy term means that it is a *conservative force*, this is important, because it makes Work Power and Energy problems involving charges much easier to do. Other *conservative forces* that we have discussed thus far include the *gravitational force* and *spring restoration force*.

What is Electric Potential/ Voltage?

Electric potential, NOT to be confused with Electric Potential Energy, is the amount of energy that must be exerted to get a charge to the point of interest (EPE) per coulomb of charge.

Hence V = $\frac{U_{electrostatic}}{q}$

Voltage is the energy analogue of Electric field, in the same way that the electric field tells you the electrostatic force per unit charge, voltage tells you the Electric Potential Energy per unit charge.

How charges behave in insulators and conductors:

The most fundamental principle to understand in order to have an understanding of how charged particles interact in insulators and conductors is that like charges repel and opposite charges attract.

In a conductor, the material has a lot of free moving electrons, meaning that they can move internally and as such they all move to the outside edges because they don't like each other and repel. This means that if you were to be locked in a metal cage in the middle of a thunderstorm and a lightning bolt hit the cage, you would be completely safe. The electrons present in the lightning bolt would repel each other and so they would only be present on the outermost skin of the cage, effectively making the cage act as a shield.

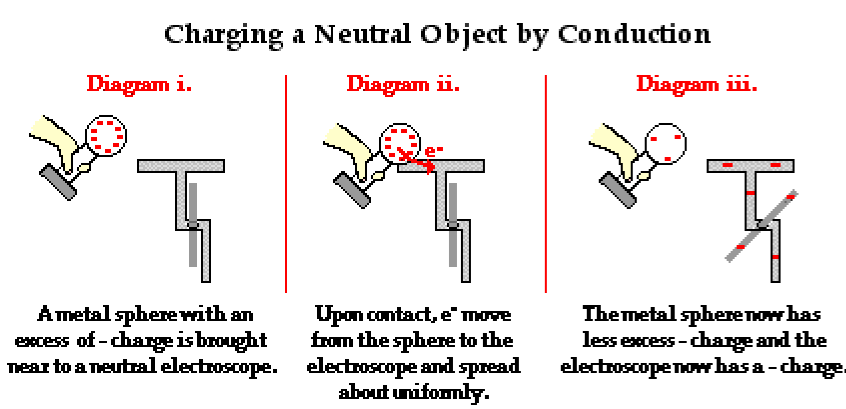
In an insulator, the material has electrons that are locked in place, they cannot move, and as such, if an insulator is charge, the charge will not be evenly distributed on the outside skin, but will instead be distributed unevenly.

Ways to charge objects and conservation of charge:

There are a myriad of ways to charge objects in real life, the list is as follows:

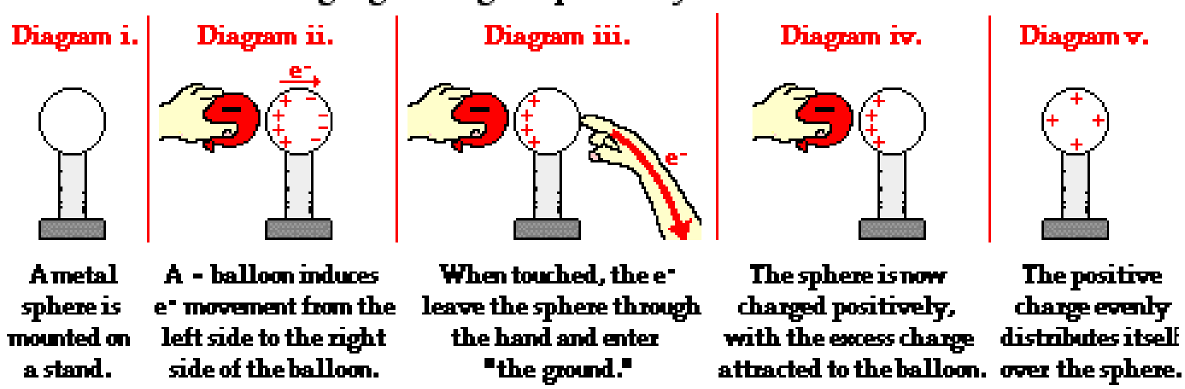
1. Charging through conduction/contact
2. Charging through induction

To charge something through contact, you must take a conducting sphere of either negative or positive charge and make it touch another conducting sphere. What this will effectively do is cause the electrons that are repelling each other on the side that makes contact to run to the other object, causing that object to be negatively charged. Likewise, if the original sphere was positively charged, the sphere that makes contact will also become positively charged. *The conservation of charge* principle states that if the original sphere had a charge of -2μC then when it makes contact with the other sphere, both will end up having -1μC. -1μC + -1μC = -2μC. See below figure to see how this happens.



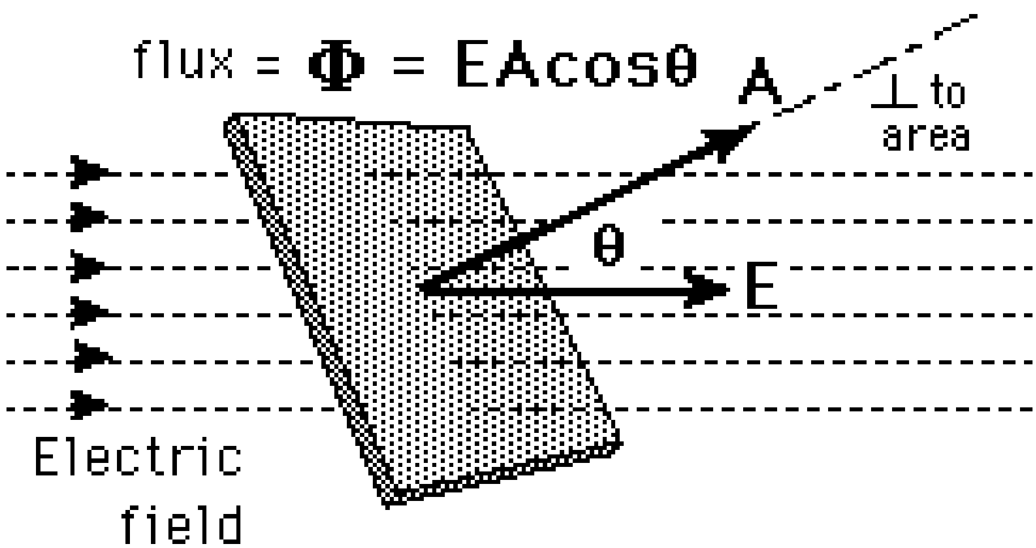
To charge something through induction, you must have a negatively charged object and put it close to a neutral object such that it is just barely NOT touching and have that neutral object be connected to ground. What this will do is cause the electrons in the object it is just barely NOT touching to run away from the excess negative charge, making them go to ground. The end result is that the formerly neutral object is now positively charged because the electrons escaped to ground. See the below diagram for a pictorial representation of this.

Charging a Single Sphere by Induction



Electric Flux:

So electric flux sounds really intimidating and complicated, it's really not. All electric flux is the number of flux lines that you draw from the charge that hit a surface in the same direction as the vector normal to the surface. See below figure for a pictorial representation of this phenomena. It is the dot product of the electric field and the area's normal vector, see the vector chapter to know what a dot product is.



The fan favorite, Gauss' Law :)

Gauss Law is a law of physics that can be used for highly symmetric scenarios to easily calculate the electric field strength(E) at a point of interest, p, remember, symmetry is your friend. Gauss came up his formulation of the law that bears his name by investigating the relationship between electric flux and three dimensional closed surfaces.

Gauss Law is expressed as a closed surface integral and the full equation is the following:

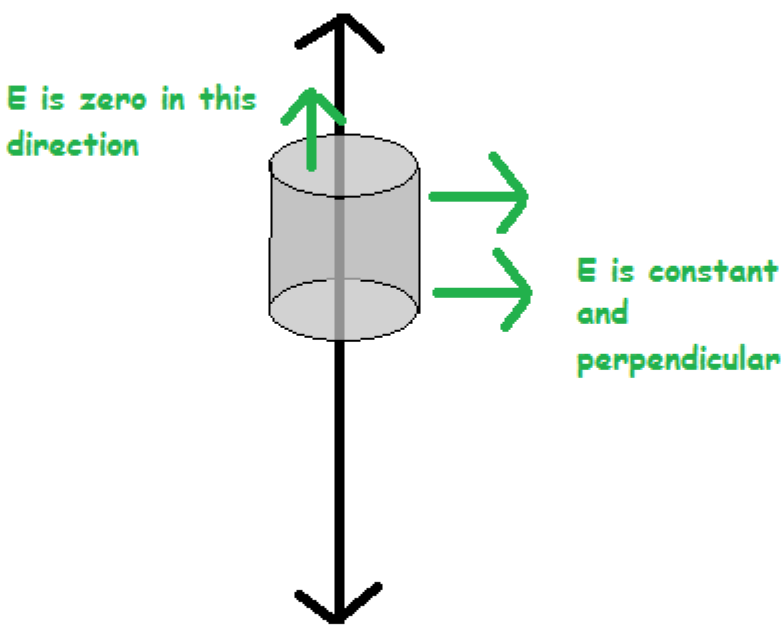
$$\Phi_{electric} = \oint E \bullet dA = \frac{q_{enclosed}}{\epsilon_0}$$

Now that looks extremely scary, like why is there an integral with a fancy circle in it and why is it a vector dot product, but luckily the dot product AND the integral will dissolve if you use this law correctly.

Conditions that must be met to effectively use Gauss Law:

1. Make an imaginary 3-D CLOSED surface (a lot of time it's a sphere or a cylinder)

2. The surface that you chose must make the field lines hit it either perfectly perpendicularly in some or all parts, or is parallel to the field lines in some or all the parts. THERE CANNOT BE A PART OF THE SURFACE THAT HAS AN ANGLE THAT IS NOT EITHER 0 OR 90 WITH RESPECT TO THE FIELD LINES.
3. The Electric field must be constant along all parts of the surface of your choice.

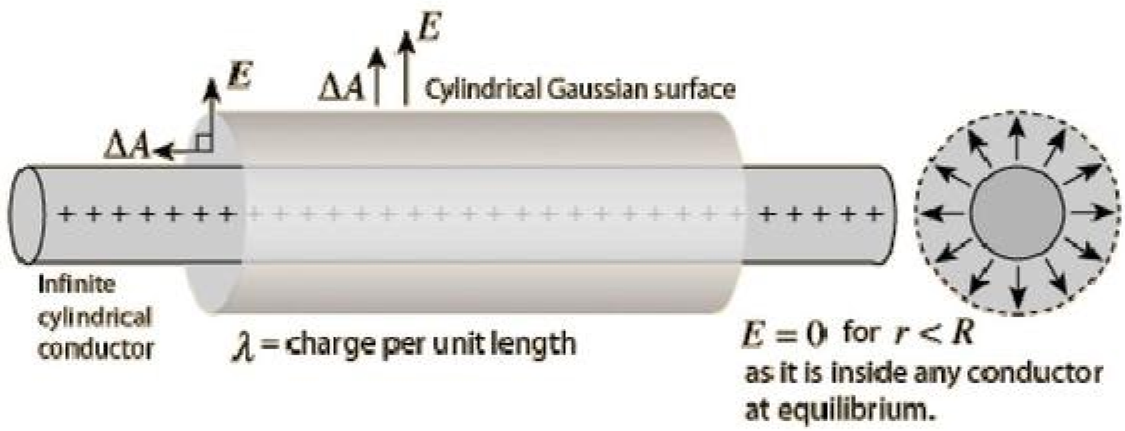


See above figure, the black line is a infinitely long positively charged rod. The Blue cylinder is the Gauss surface that we would use.

Example)

Let's suppose we have an infinitely long rod, with linear charge density λ, find the electric field strength as a function of distance, r outside the rod.

First we should draw the situation so we can visualize it and can make the appropriate Gaussian Surface.



We will make a cylindrical Gaussian surface oriented as shown above because along the curved part of the surface the field lines are penetrating it perpendicularly, while the top and bottom are parallel to the field lines. The distance from the rod is also constant from any outside point of the cylinder, therefore the E is constant.

Because the E is constant, the integral dissolves.

Because the angle that the electric field lines hit the surface are either perfectly 90 or 0 degrees, the dot product dissolves.

$$\Phi_{electric} = \oint E \bullet dA = \frac{q_{enclosed}}{\epsilon_0}$$

This equation now becomes

$$\Phi_{electric} = EA = \frac{q_{enclosed}}{\epsilon_0}$$

Now the question becomes how much charge is enclosed by the cylinder.

Well luckily for us, the rod has a uniform linear charge density.

$$\lambda = \frac{q_{enclosed}}{L}$$

The length involved in the above equation is really the height of the cylinder that we are using for the Gaussian surface so let's change that L to an h so that things will cancel nicely eventually.

$$\lambda = \frac{q_{enclosed}}{h}$$

Isolating $q_{enclosed}$ gets us the equation

$$q_{enclosed} = \lambda h$$

Because the top and bottom of the cylinder do not contribute to the electric field strength total since the field lines are parallel to the top and bottom, the areas of the top and bottom of the cylinder are not considered when computing the A value in Gauss Law in this case.

Now the question is what is the lateral area of the cylinder,

Recall that a cylinder is made up of a rectangle of height h and of base 2πr, therefore the lateral area of a cylinder is

LA = 2πrh

Plugging that in for A and plugging the other equation we got for $q_{enclosed}$ yields the following equation for Gauss Law:

$$\Phi_{electric} = E2\pi rh = \frac{\lambda h}{\epsilon_0}$$

Isolating E yields:

$$E = \frac{\lambda}{2\pi r\epsilon_0}$$

To make sure that this makes intuitive sense, let's evaluate

$$\lim_{r\to\infty} \frac{\lambda}{2\pi r\epsilon_0}$$

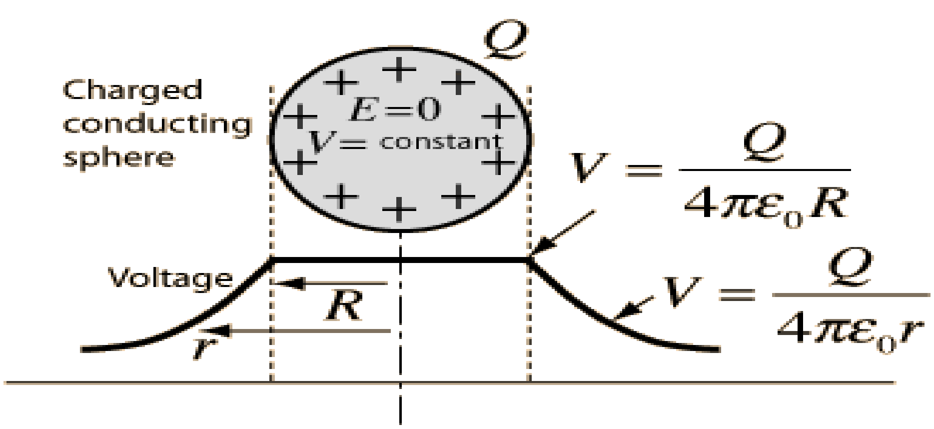
As r gets bigger and bigger, the expression approaches the value of 0, therefore,

$$\lim_{r\to\infty} \frac{\lambda}{2\pi r\epsilon_0} = 0$$

This should make some intuitive sense, because as you get farther and farther away from the charged rod the strength of the electric field *should* get weaker and weaker. It's the same reason why a charged balloon doesn't make your hair stick up if it's all the way on the other side of the room. Weak electric field = hair doesn't stick up because the electrostatic force is weak, recall $F_{electrostatic} = qE$.

Example)

Suppose we have a solid sphere of a conducting material of charge Q and radius R, find E(r).



First thing we need to do is understand that in a conducting object, on the inside the electric field is zero. This is because inside the conducting object, there is no excess charge, all excess charge is on the outermost skin. If there is no charge then there is no electric field, as charges are the source of all electric fields.

The second thing that we need to recognize is that we need to make a Gaussian Surface!

Because this is a sphere, in order for the electric field lines to hit our imaginary Gaussian Surface perfectly perpendicularly, we must make a bigger Gaussian sphere that encloses the charged sphere in question.

Now we must find out what the area of the sphere is, luckily the area of a sphere is a rather easy formula to memorize, its $4\pi r^2$, if you know the formula for volume of a sphere, you can take its derivative and then you'll get the area formula.

Gauss Law now looks like:

$$\Phi_{electric} = E4\pi r^2 = \frac{Q}{\epsilon_0}$$

Isolating E we get

$$E = \frac{Q}{4\pi r^2 \epsilon_0}$$

Recall, $k = \frac{1}{4\pi\epsilon_0}$

Therefore, E = k $\frac{Q}{r^2}$

How to calculate the E field of a solid object:

When trying to calculate the electric field strength of a solid object, we have two options:

1. Gauss Law (can only be used in highly symmetric scenarios)
2. Integrate Electric field equation derived from Coulomb's Law (hard way, but always gets you the answer)

Since we already did Gauss Law in the last section, let's do the harder way now.

Recall Coulomb's Law,

$$F_{electrostatic} = k\frac{q_1 q_2}{r^2} = q_2 E$$

Therefore,

$$E = k\frac{q_1}{r^2}$$

In a solid object, we can't just treat the whole object as a point charge because a solid object is comprised of an infinite amount of point charges that need to be added up. When we want to add up infinitely small parts, what does that remind you of? That's right! Integrals :).

Let us take a sample point charge on a charged rod, this sample point charge has a charge of magnitude dq. This small point charge has its own contribution to the total E, let us call it dE. Let's plug these values into the aforementioned equation.

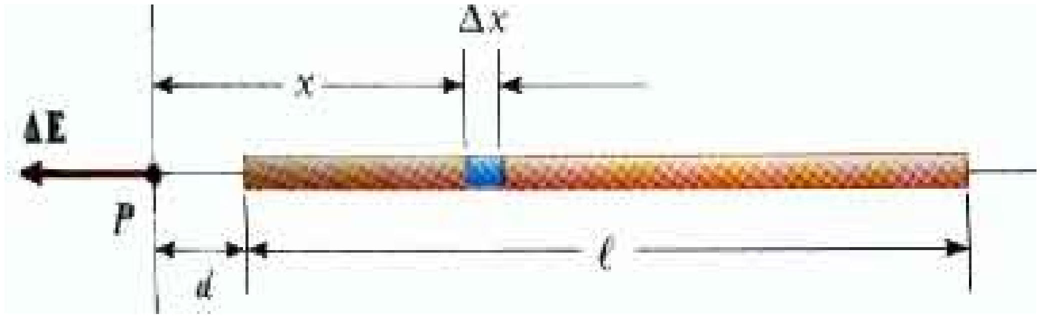
$$dE = k\frac{dq}{r^2}$$

In order for us to add up all these small contributions to the total E, we must integrate both sides.

Now the equation becomes:

$$\int_0^E dE = k\int_0^{Q_{total}} \frac{dq}{r^2}$$

Let us take a point of interest, P that is located coaxial to the rod and is located a distance d away from the right end of the rod. Find the electric field strength at point P. The length of the rod is L and the rod has a uniform linear charge density λ. See figure below to visualize it.



There is a mismatch in variables on the right hand side of the equation, this is because *each dq element is positioned at a different distance from the point of interest,* hence *the variable is r.*

Because we have *a mismatch in variables* on the right hand side of the equation, we need to find a way to transform the *dq incremental* into a *dr*. Lucky for us, and perhaps not surprisingly, they gave us the linear charge density of the rod. Recall, $\lambda = \frac{q}{r}$

Therefore, since they told us that it was a uniform density, that ratio maintains itself even at very small scales. To denote small incrementals, we will use dq to describe an infinitely small charge and dr to describe an infinitely small distance.

$$\lambda = \frac{dq}{dr}$$

Isolating dq so that we can plug into the right hand side of the integral yields the equation:

$$\lambda\, dr = dq$$

Now the integral becomes:

$$\int_{0}^{E_{total}} dE = k \int_{d}^{L+d} \frac{\lambda\, dr}{r^2}$$

Notice that the limits of the right hand side changed, this is because now we are integrating with respect to distance, rather than charge like we were originally. The lower limit is d, because the shortest distance that the dq element could be to the point of interest is d, and the upper limit is L+d because the largest distance that the dq element can be from the point of interest is L+d.

Because the linear charge density is a constant, we can take it out of the integrand.

The integral now becomes:

$$\int_{0}^{E_{total}} dE = k\lambda \int_{d}^{L+d} \frac{1}{r^2}\, dr$$

We can simplify the left hand side right now, it just becomes E_{total}

The equation is now:

$$E_{total} = k\lambda \int_{d}^{L+d} \frac{1}{r^2}\, dr$$

Now, because there is no mismatch in variables on the right hand side of the equation, we can proceed to do the integral as usual.

$$E_{total} = k\lambda \left[\frac{-1}{r}\right|_{d}^{L+d}$$

$$E_{total} = k\lambda \left(\frac{-1}{L+d} + \frac{1}{d}\right)$$

$$E_{total} = k\lambda \left(\frac{1}{d} - \frac{1}{L+d}\right)$$

Now I think you can see why we try to avoid this method as much as possible and do Gauss Law when we can…

Unit 10: Circuits

$$V = IR$$

$$P = IV$$

$$P = I^2R$$

$$P = \frac{V^2}{R}$$

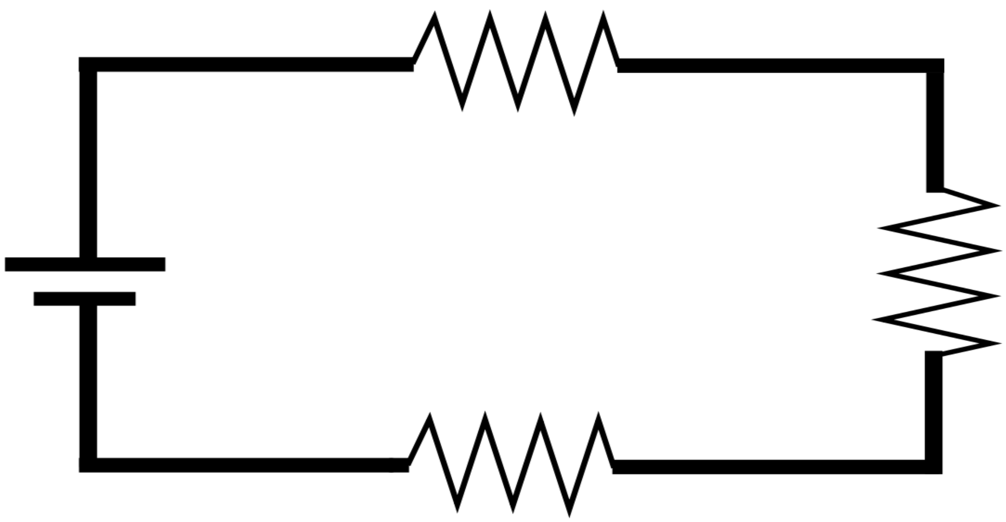
$$C = \frac{q}{V}$$

$$\varepsilon = -L\frac{dI}{dt}$$

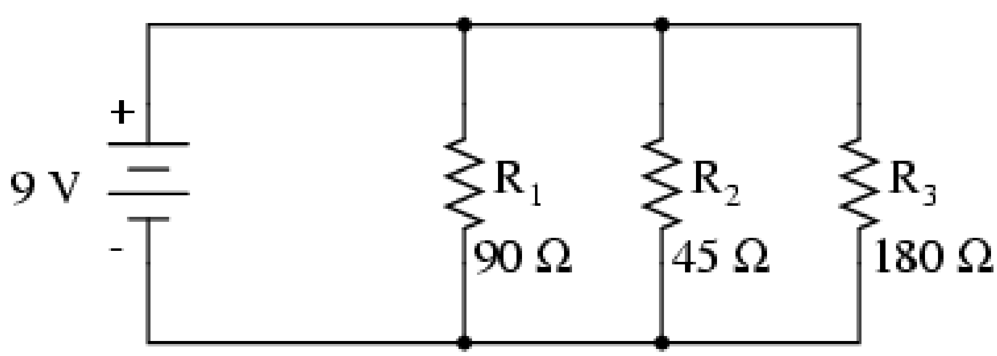
$$U_{inductor} = \frac{1}{2}LI^2$$
$$U_{capacitor} = \frac{1}{2}CV^2$$
$$U_{capacitor} = \frac{1}{2}QV$$
$$U_{capacitor} = \frac{Q^2}{2C}$$
$$R_{total\ series} = R_1 + R_2 + R_3 \ldots + R_{n-1} + R_n$$
$$\frac{1}{R_{total\ parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots + \frac{1}{R_{n-1}} + \frac{1}{R_n}$$
$$I_{circuit\ series} = I_1 = I_2 = I_3 \ldots = I_{n-1} = I_n$$
$$I_{circuit\ parallel} = I_1 + I_2 + I_3 \ldots + I_{n-1} + I_n$$
$$V_{circuit\ series} = V_1 + V_2 + V_3 \ldots + V_{n-1} + V_n$$
$$V_{circuit\ parallel} = V_1 = V_2 = V_3 \ldots = V_{n-1} = V_n$$
$$I_{in} = I_{out}$$
$$V_{battery} = V_{loop}$$
$$Q_{total\ series} = Q_1 = Q_2 = Q_3 \ldots = Q_{n-1} = Q_n$$
$$Q_{total\ parallel} = Q_1 + Q_2 + Q_3 \ldots + Q_{n-1} + Q_n$$
$$\frac{1}{C_{total\ series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots + \frac{1}{C_{n-1}} + \frac{1}{C_n}$$
$$C_{total\ parallel} = C_1 + C_2 + C_3 \ldots + C_{n-1} + C_n$$
$$I = \frac{dq}{dt}$$

What is a parallel and series circuit?

Let's start with what a series circuit is because that's the easiest, basically you can identify a series circuit by the fact that there are no branches and you can take your finger around the entire circuit without diverging. This is a series circuit, the zigzags denote resistors and the darker lines denote a battery or cell.



A parallel circuit, in contrast has branches and there are multiple paths for your finger to take. The below circuit is an example of a parallel circuit.



Sometimes you will be faced with a circuit that is a combination of both a parallel circuit and a series circuit, good news for us, those types of circuits can always be rewritten in terms of an equivalent series or parallel circuit. Then, knowing that current is constant in a series circuit or knowing that voltage in a parallel circuit is constant you can solve for any unknown. Sometimes you will have to boil down the entire circuit to a single resistor so that you can solve for the current knowing that the voltage across the single resistor is equivalent to the battery's voltage.

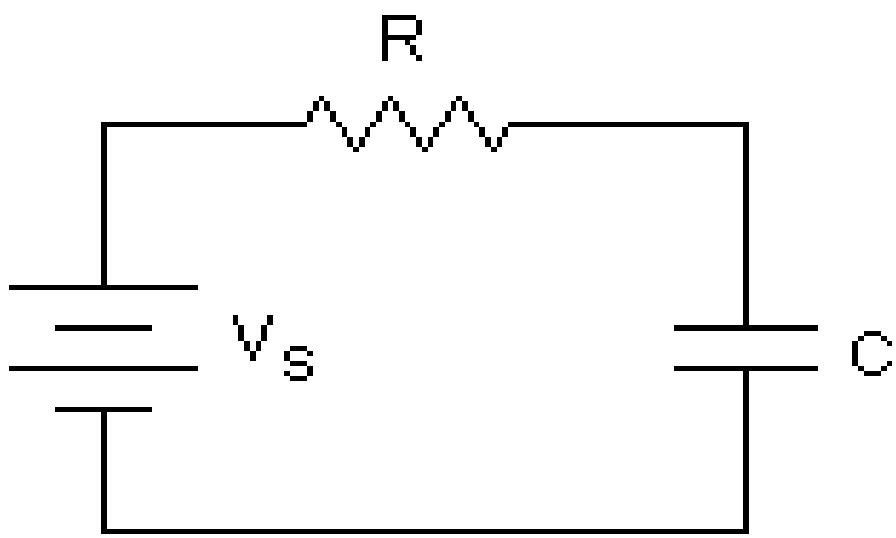
Capacitors:

Capacitors are instruments that store charge on them. When the circuit is turned on, the capacitor acts as a short (just a piece of wire), and eventually it will charge up to maximum charge. In Capacitor circuits with resistors, the same rules apply as circuits with just resistors, simplify the circuit finding equivalent resistors and equivalent capacitors and then solve for unknowns.

Sometimes we will be asked to find the equation that models the charge as a function of time for a capacitor in a circuit.

In order for us to do this, we must utilize Kirchhoff's loop rule, which states that the sum of the voltage drops in a loop equals 0. What is a loop? Well a loop is a single path of wire, it is because of Kirchhoff's loop rule that parallel resistors have equivalent voltages, each path is its own loop that connects the resistor to the battery, and because all the energy needs to be used the voltage across both resistors is the same and is the same in the battery assuming no other resistors are present in the circuit. Kirchhoff's loop rule is an extension of *conservation of energy*, all energy gained by the particles as they move through the battery needs to be used up by them as they go across the resistor, otherwise *you would end with more energy than you started with*, violating the *conservation of energy principle*.

Let us take a simple RC circuit (a circuit that contains both a resistor and a capacitor) such as the one seen below:



According to Kirchoff's loop rule:

$$\sum_{i=1}^{n} V_i = V_{battery}$$

Applying that to the above circuit:

$$C = \frac{q}{V}$$

Therefore,

$$V_{capacitor} = \frac{q}{C}$$

$$V_s = V_{resistor} + V_{capacitor}$$

$$V_s = IR + \frac{q}{C}$$

Because the current going through the circuit is not constant and varies as a function of time, we need to break down the current into its differential components.

Recall that $I = \frac{dq}{dt}$

Now the equation becomes

$$V_s = \frac{dq}{dt} R + \frac{q}{C}$$

This is a differential equation, thus to solve it we need to do what is called *separation of variables*, basically all we're going to do is take all the things with a q on one side and then put everything else on the other side. In the end we should have the q and the dq on one side and on the other side we should have dt and other variables that are NOT q.

Easiest way for us to do this effectively is to isolate $\frac{dq}{dt}$ and then put the dq where it needs to be.

$$V_s = \frac{dq}{dt} R + \frac{q}{C}$$

Let’s do that now:

$$V_s - \frac{q}{C} = \frac{dq}{dt} R$$

Now we will divide out the dq:

$$(V_s - \frac{q}{C})(\frac{1}{dq}) = \frac{R}{dt}$$

We will now take the reciprocal of both sides:

$$\frac{1}{V_s - \frac{q}{C}} dq = \frac{dt}{R}$$

We will now integrate both sides:

$$\int_0^Q \frac{1}{V_s - \frac{q}{C}} dq = \int_0^t \frac{dt}{R}$$

Notice how the upper and lower limits correspond to one another physically. At time t=0 the charge in the capacitor will be 0 C, and at an arbitrary time t, the charge in the capacitor will be some arbitrary charge Q.

Now we will have do a u-substitution in the integral,

Let u = $V_s - \frac{q}{C}$

$$\frac{du}{dq} = 0 - \frac{1}{C}$$

Isolating du yields:

$$du = \frac{-dq}{C}$$

Now we will isolate dq so that we can directly plug in.

$$-C\,du = dq$$

$$\int_0^Q \frac{1}{V_s - \frac{q}{C}}\,dq = \int_0^t \frac{dt}{R}$$

Now the above integral becomes:

$$-C\int \frac{1}{u}\,du = \int_0^t \frac{dt}{R}$$

Notice two things: firstly, we leave out the limits of integration when we plug in u and du, this is because we are now no longer integrating with respect to q. Secondly, notice that if we plug in for du and u for their expressions in terms of q and C, we would see that the integral is the exact same!

Now we will evaluate the integral for both sides:

$$-C\ [ln\left|V_s - \frac{q}{C}\right|]_0^q = \frac{t}{R}$$

Let's bring the -C to the other side now for simplification sake:

$$[ln\left|V_s - \frac{q}{C}\right|]_0^q = \frac{-t}{RC}$$

Now we will evaluate the definite integral on the left hand side:

$$ln\left|V_s - \frac{q}{C}\right| - ln\,|V_s| = \frac{-t}{RC}$$

Now we will combine the ln expressions on the left hand side:

$$ln\left|\frac{V_s - \frac{q}{C}}{V_s}\right| = \frac{-t}{RC}$$

Now we will exponentiate both sides to the power e in order to cancel the ln:

$$\frac{V_s - \frac{q}{C}}{V_s} = e^{\frac{-t}{RC}}$$

Now we will try to isolate q, because that will give us the function q(t):

$$V_s - \frac{q}{C} = V_s e^{\frac{-t}{RC}}$$

$$V_s - V_s e^{\frac{-t}{RC}} = \frac{q}{C}$$

Now multiply by C:

$$V_s C \left(1 - e^{\frac{-t}{RC}}\right) = q(t)$$

Recall,

$$C = \frac{q}{V}$$

Therefore:

$$CV = q$$

The maximum voltage that the capacitor can have across it is V_s therefore:

$$CV_s = q_{max}$$

The function then becomes:

$$q_{max} = q_{max}\,(1 - e^{\frac{-t}{RC}})$$

Now to make sure that this is the right solution and that it makes physical sense, let's evaluate $\lim_{t \to 0} q_{max}\,(1 - e^{\frac{-t}{RC}})$ and $\lim_{t \to \infty} q_{max}\,(1 - e^{\frac{-t}{RC}})$

Let's start with $\lim_{t \to 0} q_{max}\,(1 - e^{\frac{-t}{RC}})$

Plugging in 0 for t gives us this expression:

$$\lim_{t \to 0} q_{max}\,(1 - 1)$$

$$\lim_{t \to 0} q_{max}\,(0)$$

Therefore:

$$\lim_{t \to 0} q_{max}\,(1 - e^{\frac{-t}{RC}}) = 0$$

Let's now do $\lim_{t \to \infty} q_{max}\,(1 - e^{\frac{-t}{RC}})$

As t gets larger and large, $e^{\frac{-t}{RC}}$ gets closer and closer to 0 (it'll be 1/very large # which is basically 0), therefore the limit becomes:

$$\lim_{t \to \infty} q_{max}\,(1 - e^{\frac{-t}{RC}}) = \lim_{t \to \infty} q_{max}\,(1 - 0) = \lim_{t \to \infty} q_{max}\,(1) = q_{max}$$

This should make sense, because as soon as you turn on the switch the charge of the capacitor should be 0 C, but over time as you wait longer and longer (ie waiting an entire day), the capacitor should be fully charged.

Now that we have the charge as a function of time, we can find the current as a function of time as well, as I = $\frac{dq}{dt}$ so all we gotta do is differentiate our equation for q(t).

Then because we know V = $\frac{q}{C}$ all we need to do to get V(t) is just divide q(t) by C, because C does not change as a function of time.

Inductors:

Inductors are metal coils that produce a magnetic field that opposes change in current. A solenoid can be used as an inductor. When you turn on the circuit, inductors oppose all current and over time they act as a short (just a piece of wire). Basically inductors are the opposite of capacitors, because if you recall, capacitors act as shorts when you turn on the circuit and then over time they act as an open switch and oppose everything.

We can compare how inductors act with how forces act:

Recall Newton's second law:

$$F_{net} = m \frac{dv}{dt}$$

$$\varepsilon_{inductor} = -L \frac{dI}{dt}$$

In circuits and inductors, the equivalent of mass is inductance (L), in the same way that mass is physical/mechanical sluggishness, inductance is electrical sluggishness. Higher inductance = harder to change the current. Higher mass = harder to change velocity.

Recall mechanical kinetic energy:

$$K = \frac{1}{2} mv^2$$

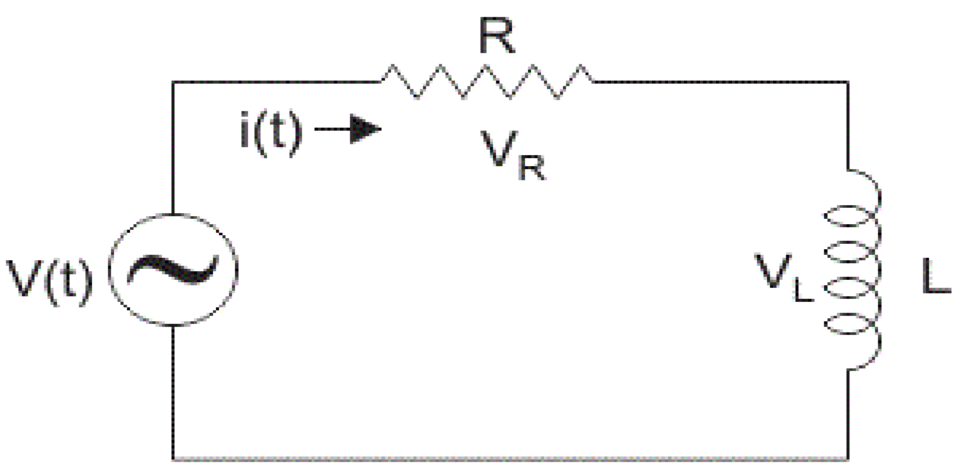
$$U_{inductor} = \frac{1}{2} LI^2$$

In summary, mass is analogous to inductance and velocity is analogous to current.

Sometimes they will ask us to find the current through the inductor as a function of time.

In order to do this, we must apply Kirchhoff's loop rule again:

Let's assume a simple RL circuit as follows:



$$V_{battery} = V_R + V_L$$

$$V_{battery} = IR + L\frac{di}{dt}$$

You may be asking why its + L $\frac{di}{dt}$ and not -, it is because as the electrons go across the inductor, they must use energy to counteract the inductor's push in the opposite direction.

This is another differential equation, we must *separate variables:*

$$V_{battery} - IR = L\frac{di}{dt}$$

Divide di on both sides:

$$(V_{battery} - IR)\left(\frac{1}{di}\right) = L\left(\frac{1}{dt}\right)$$

Take the reciprocal of both sides:

$$\frac{1}{V_{battery} - IR}dI = \frac{dt}{L}$$

Now integrate both sides:

$$\int_0^I \frac{1}{V_{battery} - IR}dI = \int_0^t \frac{dt}{L}$$

Now we need to do a u-substitution:

Let u = $V_{battery} - IR$

$$\frac{du}{dI} = 0 - R$$

$$du = -R\,dI$$

Now isolate dI

$$\frac{-du}{R} = dI$$

Substitute into integral expression:

$$\int_0^I \frac{1}{V_{battery} - IR} dI = \int_0^t \frac{dt}{L}$$

Now becomes

$$-\frac{1}{R}\int \frac{1}{u}\,du = \frac{t}{L}$$

Now multiply by -R:

$$\int \frac{1}{u}\,du = \frac{-Rt}{L}$$

Evaluate integral on left hand side:

$$[ln\left|V_{battery} - IR\right|]_0^I = \frac{-Rt}{L}$$

$$ln\left|V_{battery} - IR\right| - ln\left|V_{battery}\right| = \frac{-Rt}{L}$$

Combine ln expressions:

$$ln\left|\frac{V_{battery} - IR}{V_{battery}}\right| = \frac{-Rt}{L}$$

Exponentiate to cancel the ln:

$$\frac{V_{battery} - IR}{V_{battery}} = e^{\frac{-Rt}{L}}$$

Isolate I:

$$V_{battery} - IR = V_{battery}\, e^{\frac{-Rt}{L}}$$

$$V_{battery}\,(1 - e^{\frac{-Rt}{L}}) = IR$$

$$\frac{V_{battery}}{R}\,(1 - e^{\frac{-Rt}{L}}) = I(t)$$

Recall, V = IR, therefore I = $\frac{V}{R}$

Therefore since $V_{battery}$ is the maximum voltage, $\frac{V_{battery}}{R}$ must be maximum current.

$$I(t) = I_{max}(1 - e^{\frac{-Rt}{L}})$$

This is directly in line with what we would expect as well, because as we said in the first section on inductors, they oppose all current when you switch it on and then over time they act as a short. The limits of the function as t approaches 0 and infinity coincide with this fact.

Because the inductor and the resistor are in series, they have the same current flowing through them, therefore, we can get the voltage across the resistor as a function of time as well, by relating it through Ohm's law, or V = IR.

$$V_{resistor} = I(t)R$$

$$V_{resistor} = V_{battery}(1 - e^{\frac{-Rt}{L}})$$

This should make intuitive sense as well, because just when the circuit is turned on, there is no current going through the resistor, thus there is no voltage. Then over time the inductor becomes basically a piece of wire in the circuit, making the voltage across the resistor be the same as the voltage across the battery.

Kirchhoff's Law circuits:

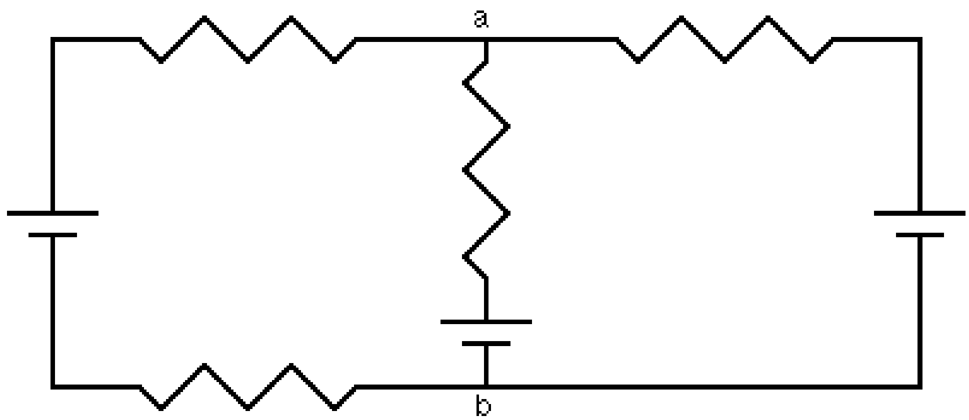
Sometimes you will be confronted with a circuit with two voltage sources (either batteries or an electrochemical cell, etc.) in this scenario you need to make use of two laws and use them to set up a system of equations in order to solve for any unknown. Be sure to pick a direction that you consider positive and negative when navigating the circuit ie clockwise or counterclockwise. If you're wrong then it will be inconsequential.

First rule:

Kirchhoff's junction rule aka the common sense rule

$$\sum I_{into\ junction} = \sum I_{out\ of\ junction}$$

This is based off of *the conservation of charge* principle, there has to be the same amount of charge leaving the junction as there is entering. A junction is where 3 or more wires converge, basically a fork in the path. A and b are junctions in the below circuit.



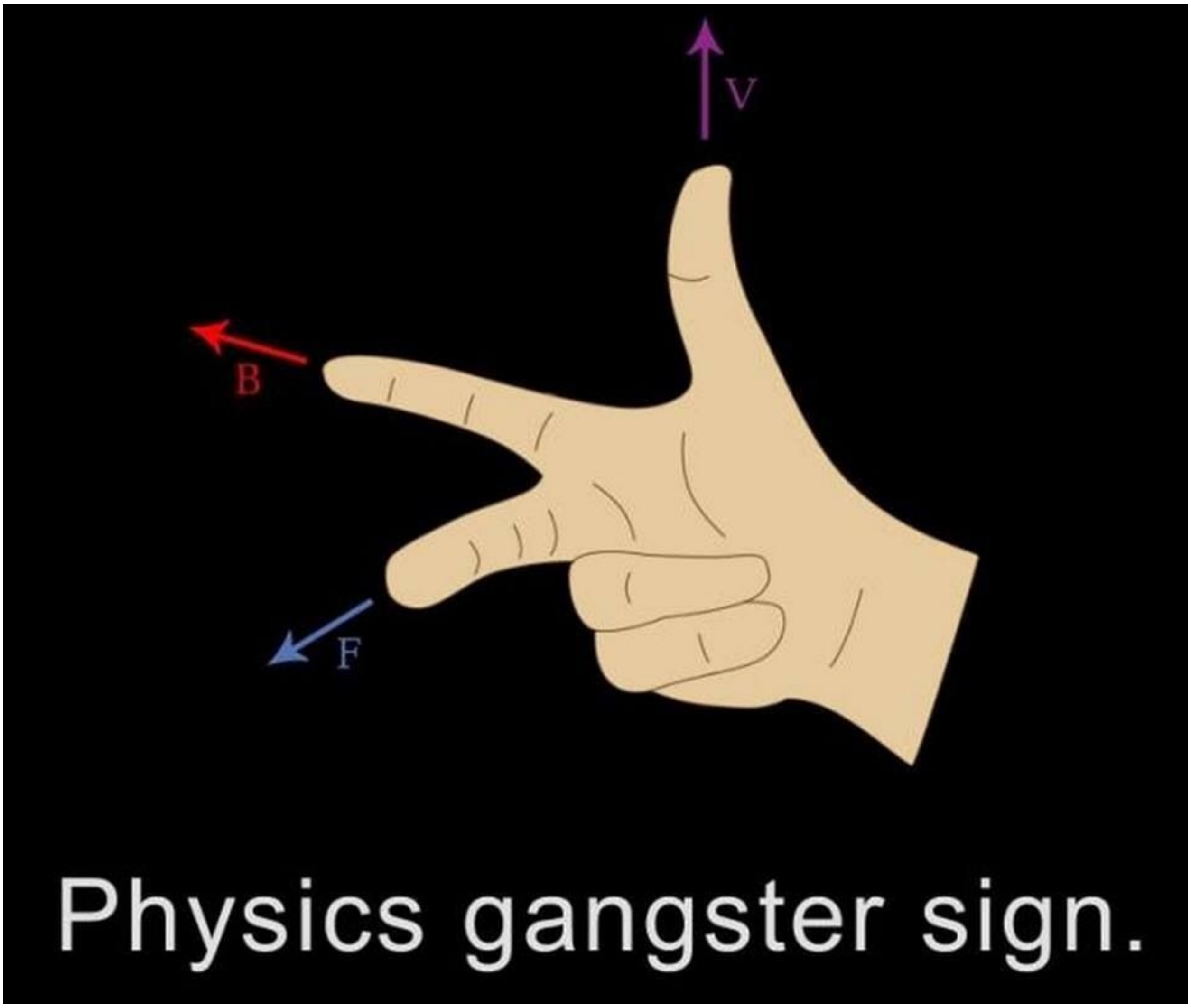
The second law that you need to be familiar with is what is known as Kirchhoff's loop rule:

$$\sum V_{loop} = 0$$

In order to use this rule you need to identify the loop you are going around and you need to identify if the particles going through the resistor or battery will gain or lose energy. If the particle goes from the negative to positive terminal then it gains energy which would mean that you would subtract that contribution, and if the particle goes from positive to negative then it loses energy and obviously if you go across a resistor then you lose energy. You're probably asking why you would subtract if you're gaining energy, it's because you're adding up the *voltage drops*, and so you're not dropping in voltage if you're gaining energy. In essence it's a "negative" voltage drop.

Using these two rules you can set up a system of equations to solve for current, voltage, resistance, etc depending on what they give you as givens. The most important thing is that these two conditions cannot be broken, otherwise charge and energy are not conserved and that'd make Kirchhoff, Professor Lewin, and Mr. Slesinski angry.

Unit 11: Magnetic forces and fields



$$F_{magnetic} = qvB\,sin\theta$$

$$F_{magnetic} = q\,(v \times B)$$

$$F_{magnetic} = ILB\,sin\theta$$

$$F_{magnetic} = L\,(I \times B)$$

$$\Phi_{magnetic} = BA\cos\theta$$

$$\Phi_{magnetic} = B \bullet A$$

$$\Phi_{magnetic} = \oint B \bullet dA = 0$$

$$\oint B\, ds = \mu_0 I_{penetrating}$$

$$dB = \frac{\mu_0 I(dl \times \hat{r})}{4\pi r^2}$$

Drawing magnetic field lines:

Magnetic fields result from the movement of charged particles, any time there is a moving charge there is also a magnetic field produced. There are other ways that magnetic fields can be produced too, such as the lining of electron domains, which are basically big chunks of electrons that are spinning around their respective atoms. When these electron domains align themselves they cause a net movement of charge in one direction, thus a magnetic field is produced.

Important things to know about magnetic field lines:

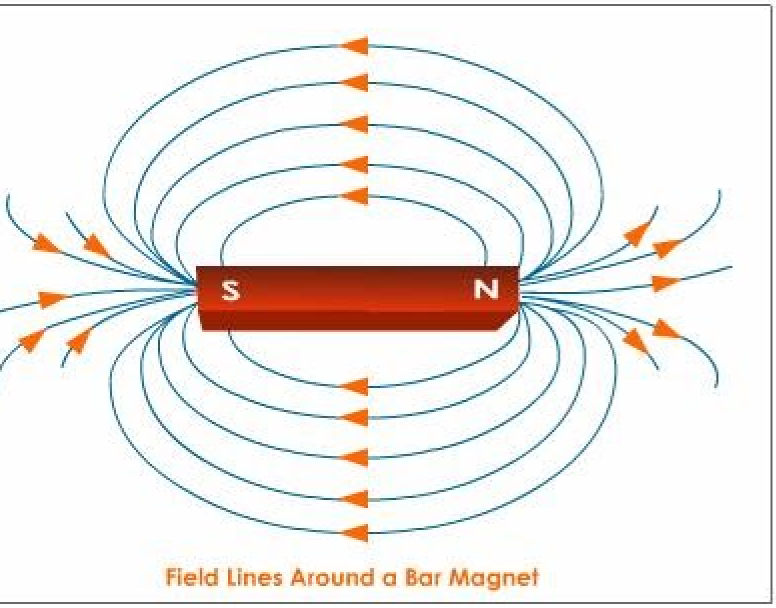
1. They flow from north to south
2. They form complete closed loops

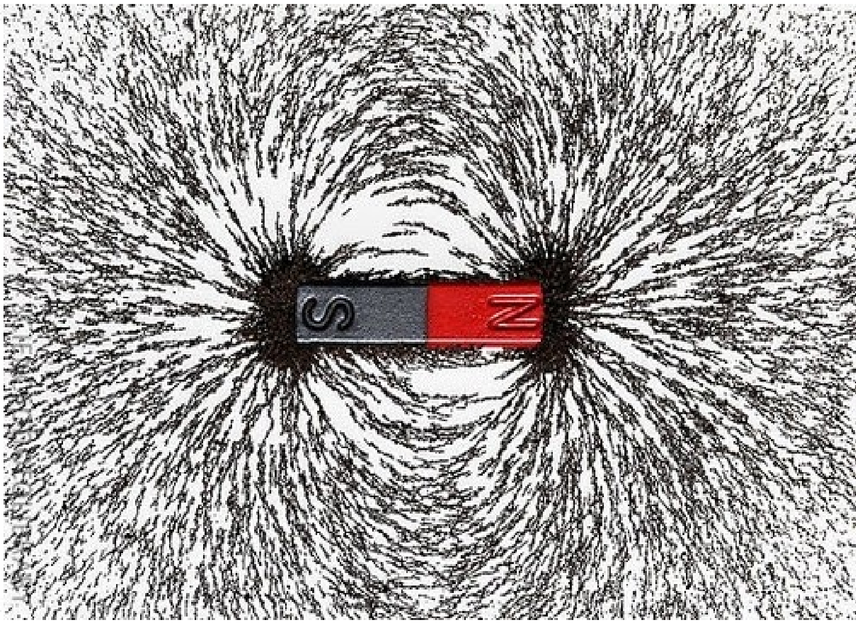
The second one is of profound importance, because to our knowledge it is impossible to make a magnetic monopole (an object with only a north pole, or only a south pole), thus we get the Gauss Law of Magnetism:

$$\Phi_{magnetic} = \oint B \bullet dA = 0$$

The amount of flux lines going into any closed surface around a magnet is always equal to the amount of flux lines leaving the closed surface, because they form closed loops.

In order to draw magnetic field lines, one must pretend to place an imaginary north pole/compass at a given point and see which way it would orient itself.





The Magnetic Force:

A magnetic force is the force that a charged object experiences when exposed to an external magnetic field. The magnetic force is expressed as the cross product of the velocity vector and the magnetic field (B) vector. However, the magnetic field is also dependent on the charge of the object in question, the object must be charged in order for the magnetic field to have any effect on the object. Because it is expressed as a cross product of the velocity and magnetic field vector, the magnetic force is always perpendicular to both the velocity and the magnetic field. This is important, because recall that W = $F \bullet x$ this means that because the force is perpendicular to the velocity it is also perpendicular to the displacement, thus the magnetic force DOES NO WORK!! In order to determine the direction of the magnetic force, we need to use the right hand rule, see cross product chapter for more details.

In a situation with two wires that have steady currents flowing through them, if their magnetic fields point in the same direction, they will attract, and if their magnetic fields point in the opposite direction, they will repel.

Magnetic Field visualization rules:

There are a couple of things you need to keep in mind to help visualize the magnetic fields of objects that go beyond simply a bar magnet.

For a current carrying wire, the magnetic field is a doughnut shape around the wire, and the direction of it is found using the following technique:

Pretend you're grabbing the wire itself, with your thumb pointing in the direction of the current. The way your fingers are curved is the direction of the magnetic field around the wire.

For solenoids, the magnetic field will be the same as a bar magnet's, however, in order to determine direction you need to use the following technique:

Pretend you're grabbing the solenoid and align your fingernails with the direction of the current in the wire, the end that your thumb is pointing is the north pole of the magnetic field.

Ampere's Law:

Because Gauss' law of magnetism is not useful in calculating the magnetic field, physicists needed to come up with some other way that would allow us to easily calculate the magnetic field given known variables. Instead of a closed surface integral that we used in Gauss' Law, Ampere's Law makes use of a closed line integral.

Ampere's Law is formally stated as:

$$\oint B\, ds = \mu_0 I_{penetrating}$$

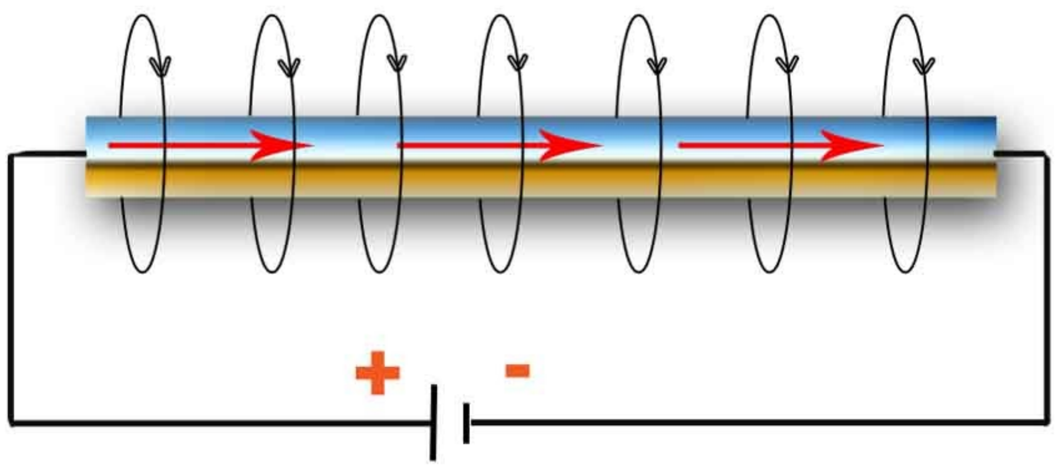
In order to use Ampere's Law we must remember the following:

We must *draw out the scenario* and make sure that we draw our Amperian surface such that at any point along the ring/ two-dimensional shape the *magnetic field is constant* (at the *same distance away* from the object of interest).

Then the equation will function the same as Gauss' Law of electricity, except instead of surface area, we will use circumference/ length (this is due to the fact that we are working with a *closed line integral* instead of a *closed surface integral* and thus we are integrating with respect to *distance, not area*).

Example 1)

Suppose we have a wire carrying a steady current of magnitude I, find the function B(r) where r is the distance away from the wire. The function will only be defined for points outside of the wire itself.



Because we know that the magnetic field around a current carrying wire is shaped like a doughnut around the wire itself, we know that if the amperian surface is made such that each point is the same radial distance away from the wire the magnetic field would be constant. This is why we must draw a figurative loop around the wire and that will be our Amperian loop for this example.

Start out by writing out Ampere's law:

$$\oint B\, ds = \mu_0 I_{penetrating}$$

Because we drew the amperian loop such that it is concentric with the current carrying wire, the current that is penetrating the Amperian loop is of magnitude I, the same as the current in the wire.

Because the magnetic field is constant around the Amperian loop, the B can be taken out of the integrand.

The equation now becomes:

$$B \oint ds = \mu_0 I_{penetrating}$$

The closed integral of ds is the circumference of the circle, now the equation becomes:

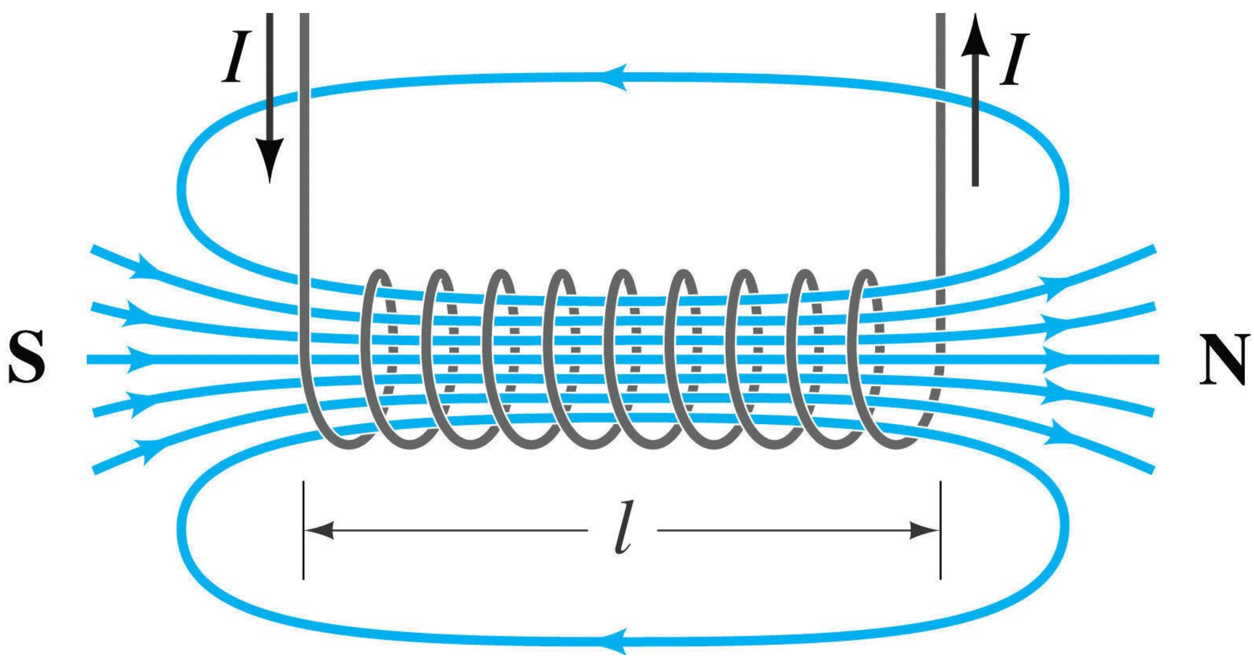
$$B\, 2\pi r = \mu_0 I_{penetrating}$$

Isolating B yields the basic function:

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

Example 2)

Now suppose we have a solenoid of length L with N turns. Calculate the magnetic field strength inside the solenoid.



In order for us to calculate the magnetic field strength of the solenoid, we must use Ampere's law.

First step whenever we do an Ampere's law problem is to identify what our Amperian surface will be.

Because we know that the magnetic field around a solenoid is the same as that of a bar magnet, we know that the distance away from the center of the solenoid is the only thing that affects magnetic field strength. Moving along the axis of the solenoid makes no difference in magnetic field strength because the flux per unit area is the same anywhere along the axis of the solenoid.

Because of this we are going to make a fictitious rectangle such that the rectangle extends from one end of the solenoid to the other and extends outward.

This Amperian surface is a bit more complicated than the previous one because we need to split up the closed surface integral into separate integrals to deal with each side of the rectangle. This is because the magnetic field along each part of the Amperian surface is not uniform, and thus needs to be integrated separately. The rectangle will have sides, a, b, c, and d. The only side that will be inside the solenoid will be side d.

Start out by writing out Ampere's law:

$$\oint B\,ds = \mu_0 I_{penetrating}$$

To break this closed line integral up we need to do the following:

$$\int B_a \, ds + \int B_b \, ds + \int B_c \, ds + \int B_d \, ds = \mu_0 I_{penetrating}$$

Luckily for us, this boils down quite nicely. This is because anywhere outside the solenoid the magnetic field is so weak that is does not contribute enough to the overall magnetic field to be calculated. This means that $B_a, B_b, and \, B_c$ are all 0 in magnitude, thus all of those integrals cancel out because 0 times anything is 0.

The equation then becomes:

$$\int B_d \, ds = \mu_0 I_{penetrating}$$

The side d extends from 0 to the full length of the solenoid, L and the magnetic field is constant along the axis of the solenoid, thus the equation becomes:

$$B_d \int_0^L ds = \mu_0 I_{penetrating}$$

Now that we have a fairly straight forward integral, we now need to figure out an expression of the magnitude of the current that is penetrating the Amperian surface. Because it is a solenoid, there is a current penetrating the surface at every point there is a loop or turn in the solenoid, therefore the total current penetration is dependent on the number of loops in the wire.

$$I_{total} = N I$$

Now the equation becomes:

$$B_d L = \mu_0 I_{penetrating}$$

Isolating magnetic field strength yields the following:

$$B_d = \frac{\mu_0 N I}{L}$$

Loop density n = $\frac{N}{L}$

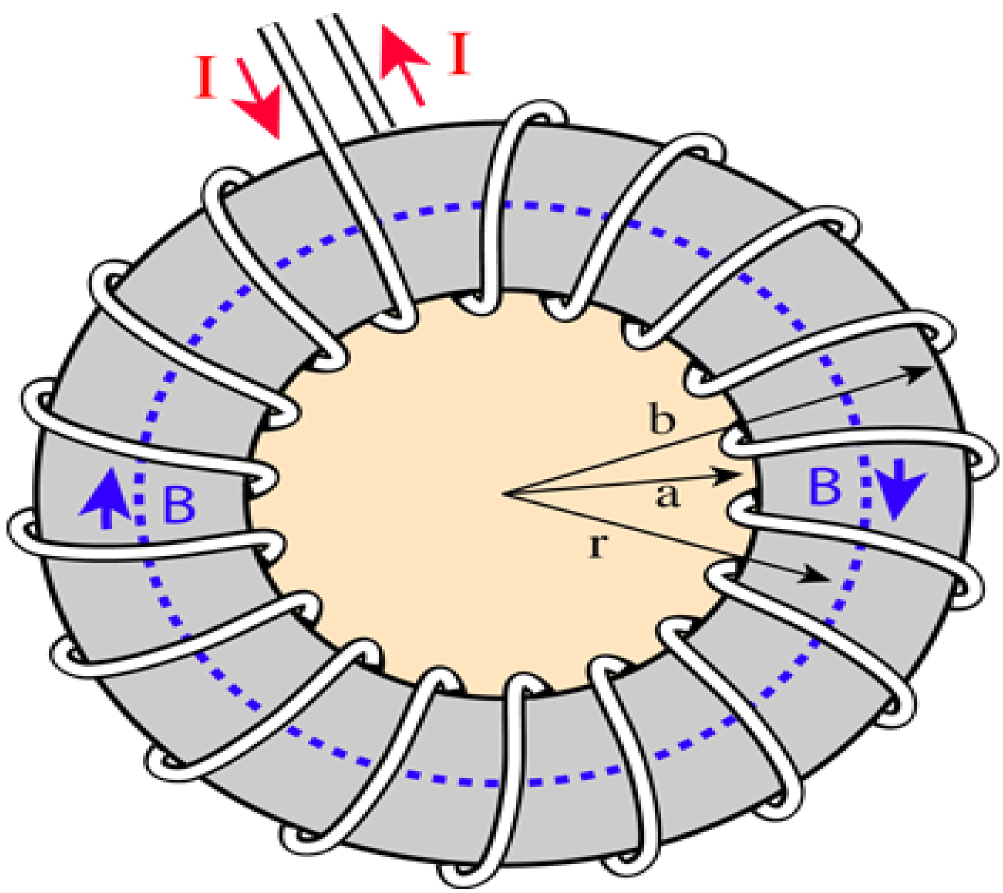
Therefore:

$$B_d = \mu_0 n I$$

Example 3)

A toroid is a solenoid, but in the shape of a circle. Find the magnetic field strength inside the toroid if the current flowing through the wires is of magnitude I and the toroid has N number of loops.

Because a toroid is basically a circular solenoid, we know that inside the toroid the magnetic field is more or less uniform, because of this, we know that we can make an amperian loop around the circumference of the toroid to calculate the magnetic field strength.



The figure shown above models exactly what we're going to do to calculate the magnetic field of the toroid.

Start by writing Ampere's Law:

$$\oint B \, ds = \mu_0 I_{penetrating}$$

Because everywhere along the amperian loop we drew the B is constant we don't have to break the closed line integral up into separate integrals like we had to do in the last example. We can take B out of the integrand yielding the following equation:

$$B \oint ds = \mu_0 I_{penetrating}$$

The surface we chose is circular in nature, and thus the integral of ds is the circumference of the circle yielding:

$$B \, 2\pi r = \mu_0 \, I_{penetrating}$$

I penetrating is the number of loops times the current:

$$B \, 2\pi r = \mu_0 \, N \, I$$

Solving for B yields the following equation:

$$B = \frac{\mu_0 N I}{2\pi r}$$

Magnetic Flux:

Magnetic flux is the exact same as electric flux, but with magnetic field lines instead of electric field lines. Please refer to electric flux chapter for more details.

Biot-Savart's Law:

In the same way that in electricity we sometimes had to integrate Coulomb's law in order to get E, in magnetism where situations are not symmetric enough for you to use Ampere's Law, Biot-Savart Law is used. If you have the option of using Ampere's Law, PLEASE USE AMPERE'S LAW, DO NOT USE THIS LAW UNLESS YOU ABSOLUTELY 100% HAVE TO!!

Biot and Savart wanted to make an equation that related an infinitely small piece of the magnetic field to the current, the permeability of free space, and distance. What they found was that they had to break up the current into what are referred to as "current elements" which are basically little tiny pieces of the wire that carry the current.

Biot-Savart Law is formerly written as:

$$dB = \frac{\mu_0 I \, dl \times \hat{r}}{4\pi r^2}$$

μ_0 is the permeability of free space,
I is the current in the wire
dl is the infinitely small piece of the wire

$\hat{r}$ is the unit vector that points from the current to the magnetic field point
r is distance

In order to show that this law isn't bogus and does actually work we're going to do an example that we did using Ampere's Law to prove that 1, this law does in fact work and 2, that Ampere's Law is much easier.

Let's start out by having a current carrying wire carrying current I, find B(r)!

Let's start out by looking at the point of interest, which is at an arbitrary distance r away from the current carrying wire.

Let's now write down Biot-Savart's Law:

$$dB = \frac{\mu_0 I \, dl \times \hat{r}}{4\pi r^2}$$

First thing we want to do is get rid of that pretty disgusting looking cross product. We can do this by replacing the cross product with $\sin\theta$.

$$dB = \frac{\mu_0 I \, dl \, sin\theta}{4\pi r^2}$$

Well the sin of theta is rather difficult to look at, because the angle changes. The perpendicular distance away from the wire will be denoted as R, while r refers to the distance the point of interest is away from the dl incremental. The distance across the wire will be denoted as l

$$sin\theta = \frac{R}{r}$$

$$r = \frac{R}{sin\theta}$$

Because the angle between the dl incremental and the point of interest changes, we need to integrate over theta.

$$tan\theta = \frac{-R}{l}$$

$$l = -Rcot\theta$$

Differentiate in order to get relation between dl and dθ yields

$$\frac{dl}{d\theta} = R \, csc^2\theta$$

Therefore,

$$dl = R\, csc^2\theta\, d\theta$$

Recall that $csc\theta = \frac{1}{sin\theta}$

Therefore:

$$dl = \frac{R}{sin^2\theta} d\theta$$

We want to write it in terms of sin so that things can cancel out when we plug back into the integrand.

The equation now becomes:

$$dB = \frac{\mu_0\, I\, R}{4\pi(\frac{R^2}{sin^2\theta})sin\theta} d\theta$$

$$dB = \frac{\mu_0\, I\, sin\theta}{4\pi\, R} d\theta$$

Now we will integrate both sides:

$$\int_0^{B_{total}/2} dB = \int_0^{\pi/2} \frac{\mu_0\, I\, sin\theta}{4\pi\, R} d\theta$$

Notice that the limits of the left side is ½ the total B, this is because we are only looking at one half of the rod with the limits of integration we have on the right side.

$$\frac{B_{total}}{2} = \frac{\mu_0 I}{4\pi R} \int_0^{\pi/2} sin\theta \, d\theta$$

$$B_{total} = \frac{\mu_0 I}{2\pi R} [-cos\theta]_0^{\pi/2}$$

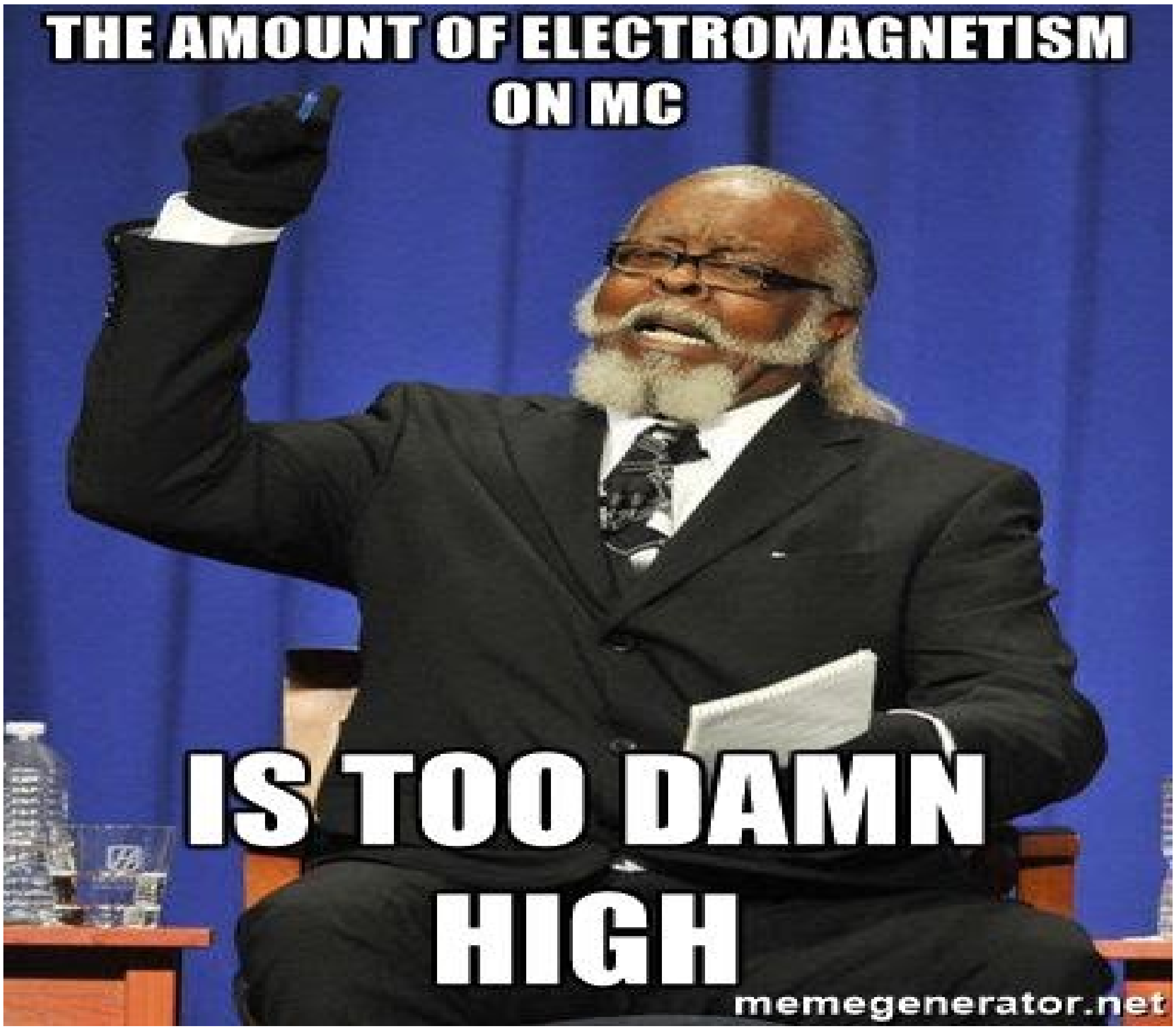
$$B_{total} = \frac{\mu_0 I}{2\pi R} (-cos(\tfrac{\pi}{2}) + cos(0))$$

$$B_{total} = \frac{\mu_0 I}{2\pi R} (0 + 1)$$

$$B_{total} = \frac{\mu_0 I}{2\pi R}$$

Now you understand why we try to avoid this law as much as humanly possible. Ampere's law is much easier.

Unit 12: Electromagnetic Induction



$$\varepsilon_{induced} = N\frac{-d\Phi_{magnetic}}{dt}$$

$$\Phi_{magnetic} = \int B \bullet dA$$

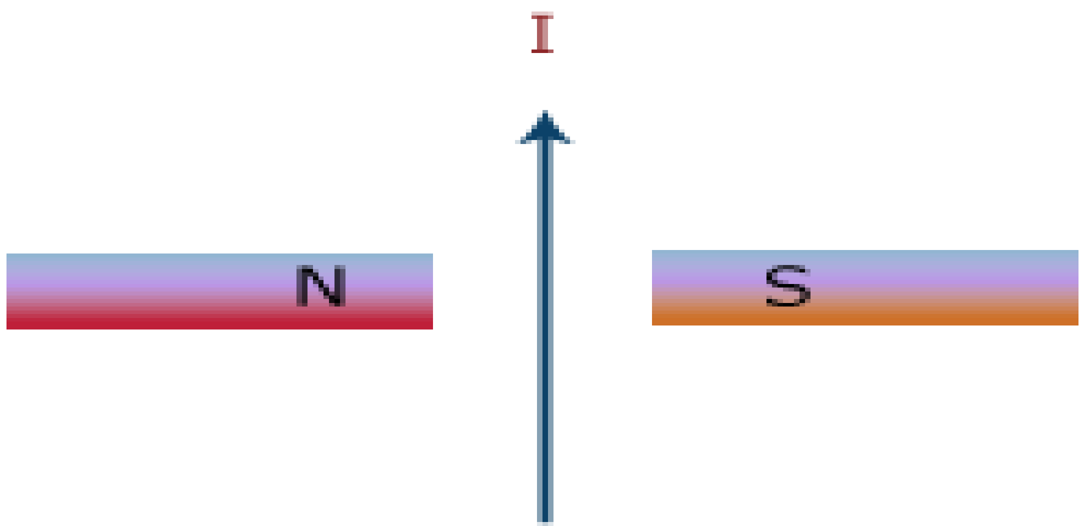
$$\varepsilon_{induced} = \frac{-d(\int B \bullet dA)}{dt}$$

$$\varepsilon = \oint E\, dl$$

The basic interactions:

Magnetic forces act on wires, in addition to them producing their own magnetic field, when wires are exposed to an external magnetic field produced by bar magnets, they experience a magnetic force.

See the below figure:



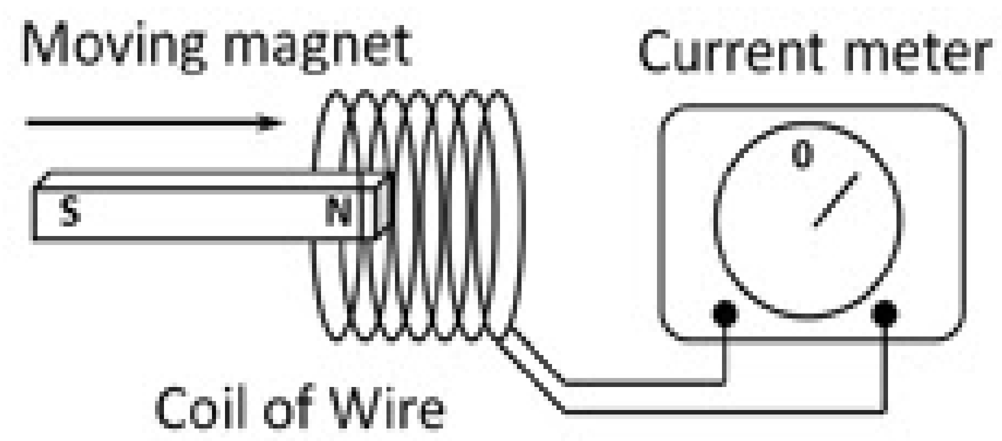
Recall that magnetic field lines flow from N to S, therefore there is a net magnetic field to the right in the above diagram.

Recall that current is simply a flow of charge, charged particles that have a velocity. Remember that the magnetic force is expressed as a cross product of the velocity vector and the magnetic field vector. Therefore the force vector is directed into the page, thus the wire accelerates into the page.

Now let's assume that the wire does not have any current flowing in it, would there be a magnetic force being exerted on the wire? Well *the wire has latent charge*, due to its electrons, but because *the wire does not have a current flowing*, the electrons are not moving. Therefore, *we need to move the wire* in order to cause the electrons to move with respect to the magnetic field produced by the bar magnets *in order to produce a magnetic force*. Now the charges have a velocity, thus if we moved the wire into the page, the magnetic force would be directed upward. This magnetic force will now affect the charges in the wire, *inducing a current in the wire in the direction of the magnetic force.*

Lens' Law:

Because of the observation we discussed above about the moving wire in the presence of a magnetic field, scientists wanted to see if moving the magnetic field would cause an induced current in a wire. It was found that moving a magnet away from a coil of wire caused a current to be induced in the wire such that the magnetic field produced in the wire would be such that it would attract the moving magnet. The magnetic field was experimentally determined to always oppose the motion of the original magnet. This is what Lens' Law is at its core.



Example) see above figure.

Because the magnet is being plunged into the coil of wire, the current induced in the coil of wire would cause a north pole to form in the left side of the coil so as to oppose the motion of the bar magnet being plunged into the coil of wire. The current direction can be found by using the solenoid current right hand rule discussed in previous chapters. The thumb points in the direction of the magnetic north pole and the finger tips curl in the direction of the current.

Faraday’s Law:

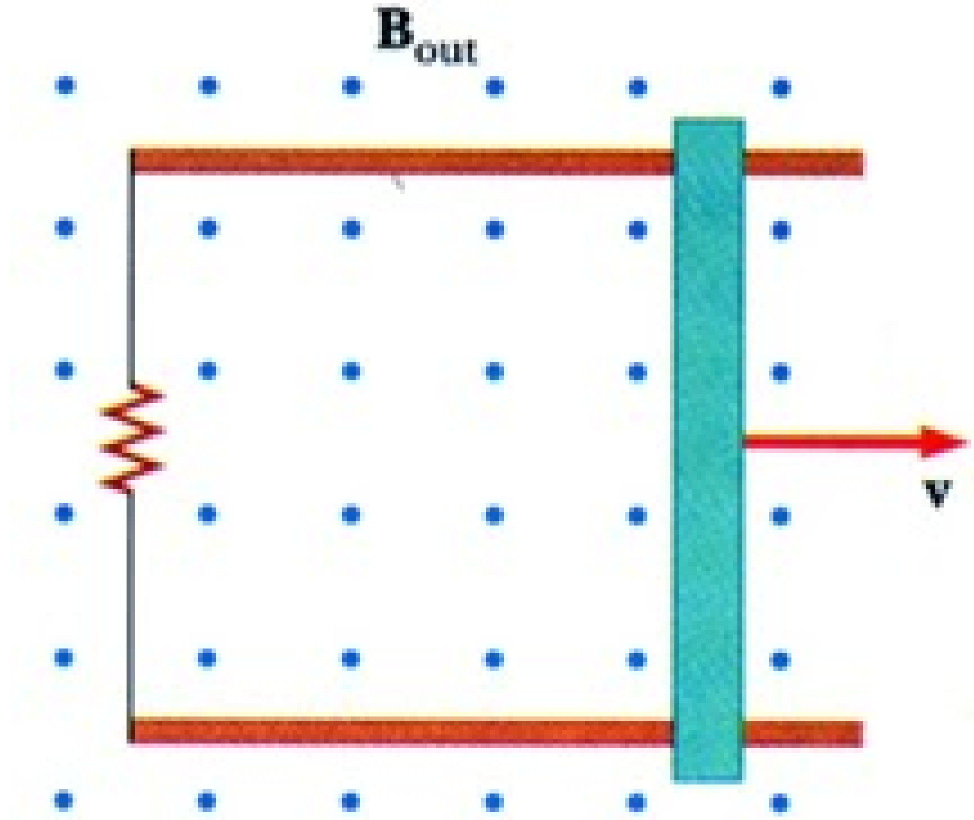
Michael Faraday wanted to see if a changing magnetic field could induce a voltage in a conductor. His work involved analyzing the effect of the rate of magnetic flux busting on induced voltage. What he found was that the faster the magnetic flux is busted, the higher voltage that is induced in the conductive material. He also determined that the number of loops of the conductive material also contributed equally to the induced voltage.

Faraday’s Law is formally written as:

$$\varepsilon_{induced} = -N \frac{d\Phi}{dt}$$

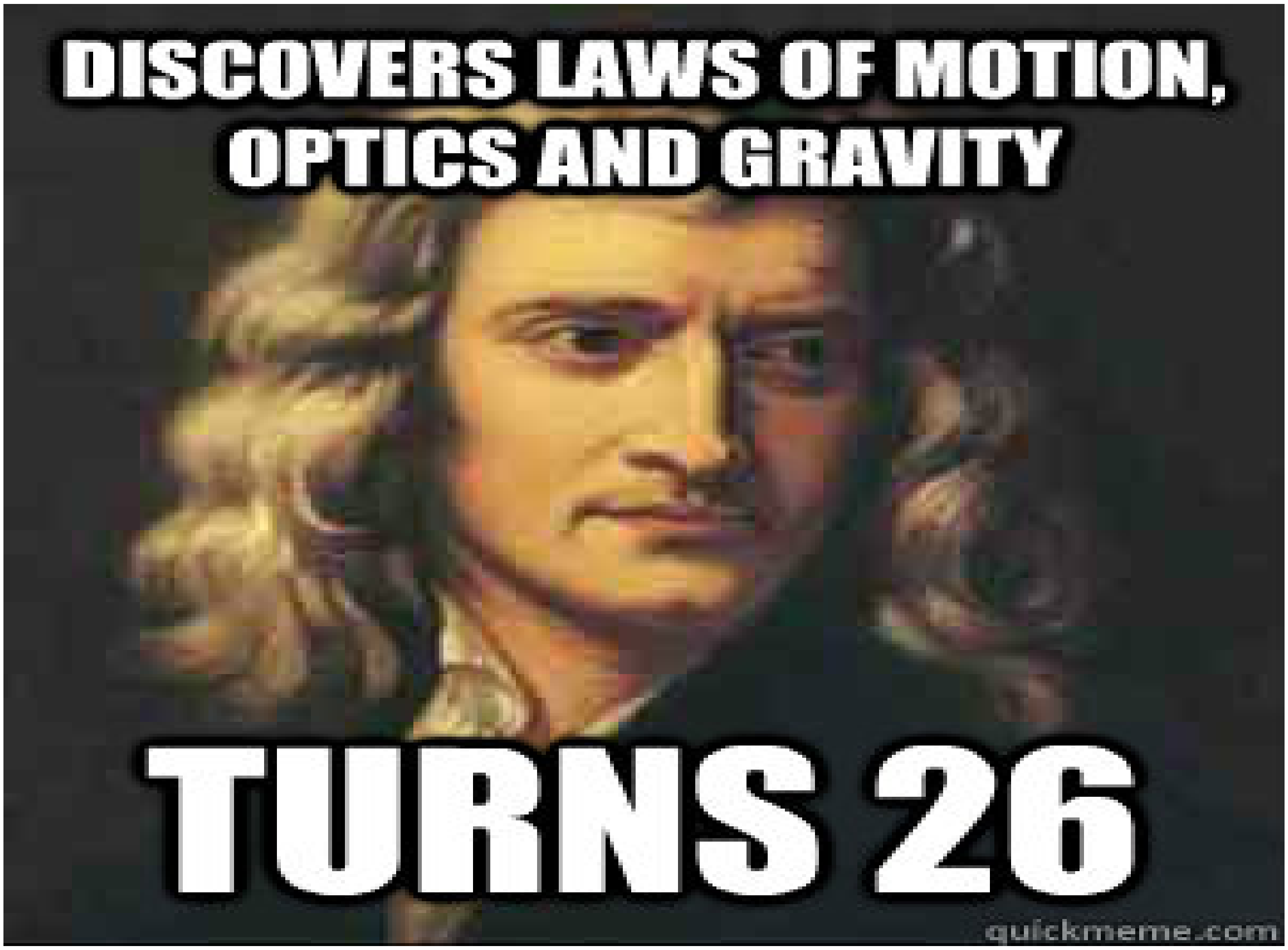
The reason that there is a negative there is because the potential difference is in the direction opposite that of the magnetic flux busting.

It was found that steady electric fields/currents produce magnetic fields, but steady magnetic fields DO NOT produce electric fields. In order for there to be any induced current or voltage, THERE MUST BE A CHANGE IN THE MAGNETIC FIELD. In other words, the flux needs to be busted.



Because the above bar is going through a magnetic field, there will be an induced voltage across the bar.

Chapter 13: Optics



$n_1 sin(\theta_1) = n_2 sin(\theta_2)$

$n = \frac{v}{c}$

$\theta_{incident} = \theta_{reflected}$

$v = f\lambda$

$E_{individual\ photon} = hf$

$E_{individual\ photon} = h\left(\frac{c}{\lambda}\right)$

$For\ photoelectric\ effect,\ KE_{electron} = h\nu - \varphi_{binding\ energy}$

$$\lambda_{matter} = \frac{h}{mv}$$

Quantum Physics:

At the turn of the century, it was thought that all physics was discovered and that everything was deterministic in nature. In other words, everything that happens in the universe can be predicted with a high degree of accuracy using classical physics. However, we know now that classical physics do not hold in the nanoscopic scale and with objects moving close to the speed of light.

Albert Einstein, though against Quantum Physics at its inception and fought to disprove it, has two postulates for his theory of general relativity that go against classical physics:

1. There is no preferential reference frame when dealing with objects approaching the speed of light (the perspective does not matter)
2. Nothing can move faster than the speed of light, and the speed of light is constant irrespective of the medium through which it travels.

It was also discovered that photons behave in an irregular fashion, sometimes they behave like particles and other times they behave like waves. This behavior depended on the experiment done. We will discuss the wave and particle duality in greater detail later. It was later discovered that electrons behave like particles and waves as well.

Photoelectric Effect:

What baffled scientists for decades is the phenomena called the Photoelectric Effect, this occurs when light is held incident on a piece of metal and electrons come flying off the metal. Classical physics would dictate that as the intensity of the light increases, electrons would come flying off faster, this is because the amplitude of the light wave is related to its intensity and with a greater amplitude the energy of the light wave should increase. This however, was not the case. It was found by Albert Einstein that the kinetic energy of the electron was related not by the amplitude of the light wave, but rather by its frequency. He postulated that light is comprised of photons, little packets of light all traveling at the speed of light infinitely close together and that their energy was related by the equation:

$$E = hf$$

He found that there is a certain frequency that must be reached in order for electrons to fly off the metal, this frequency is called the threshold frequency. Einstein stated that the reason why this exists is because there must be a certain amount of energy that must be absorbed by the electron in order to break the electrostatic force of attraction between the electron and the protons present in the atoms of the metal slab. This energy that needs to be broken is called the binding energy and is denoted as φ. The resultant kinetic energy of the electron can be tabulated by relating it to the binding energy and the energy of the incident photons. The equation is:

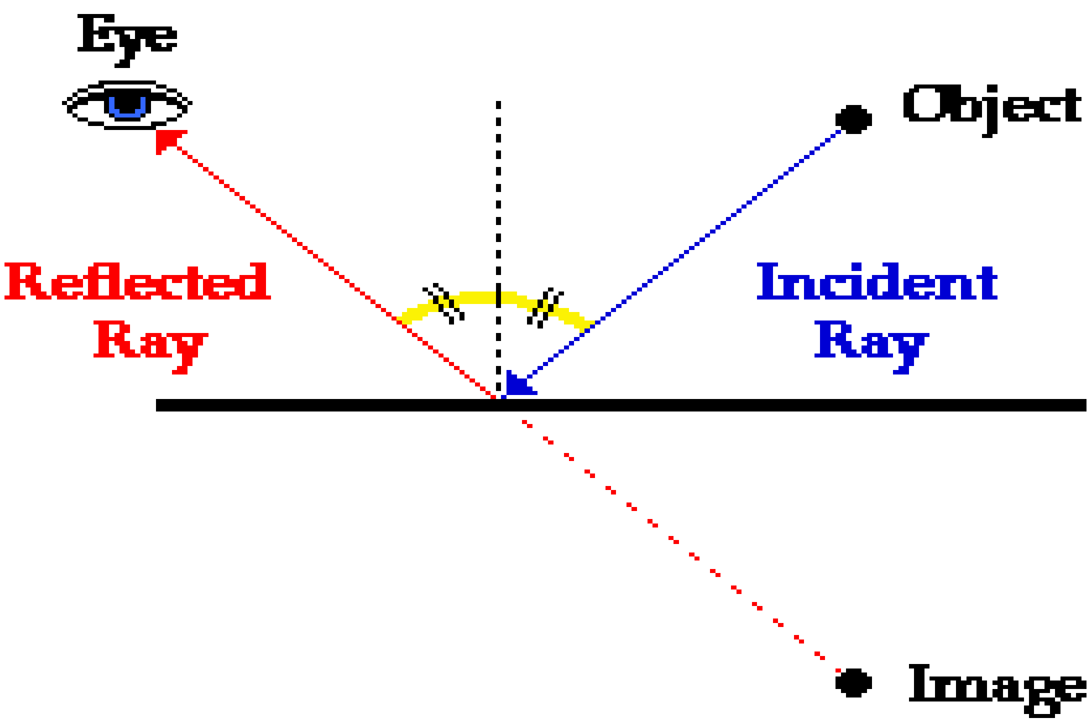
$$KE_{electron} = h\nu - \varphi_{binding\ energy}$$

Once the threshold frequency is reached, the intensity of the light determines how many electrons are flying off the metal slab. The higher the intensity of the light, the more electrons that fly off per second. However, regardless of the intensity of the light, if the threshold frequency is not reached, NO electrons will fly off.

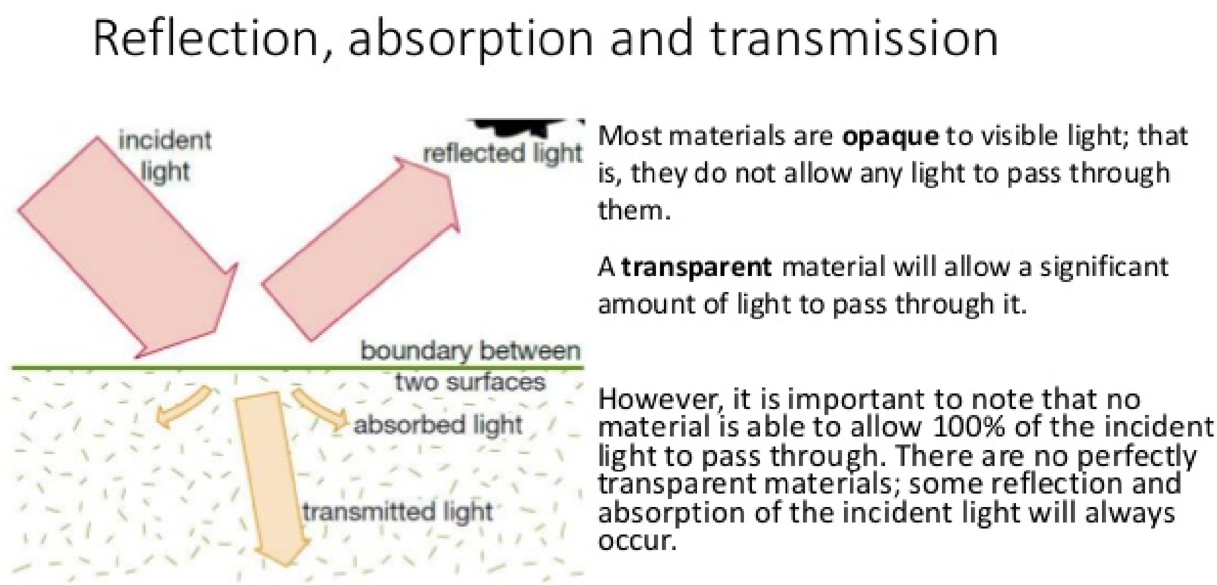
It is also important to note that when the energy of the incident photons are equivalent to the binding energy, the resultant kinetic energy of the electron is zero J. Remember that $KE = 1/2\,(mv^2)$ so the electron will fly off faster when hit with higher energy incident light.

Reflection, Refraction, and Diffraction:

Whenever light hits an object, 4 things happen, the light is reflected, transmitted/absorbed, refracted, or diffracted. Reflection occurs when the light “bounces” off the object in question. The angle at which the light is incident on the object with respect to the normal is equal to the angle at which the light is reflected on the object with respect to the normal. See figure 13.1 to elucidate this:



Transmission and absorption is defined as when light goes through an object or its energy is captured by that object. See figure 13.2 to elucidate this point:



Refraction is when light is bent due to differing mediums, for example when light goes from air to water, the light is bent, or refracted. This phenomenon is also responsible for the separation of light wave frequencies (colors) when white light is incident on a prism. Different frequencies of light have different coefficients of refraction, n, and therefore bend to different degrees, giving us the distinct rainbow we see when light is incident on the prism. This relationship is dictated by Snell's Law, this equation states that:

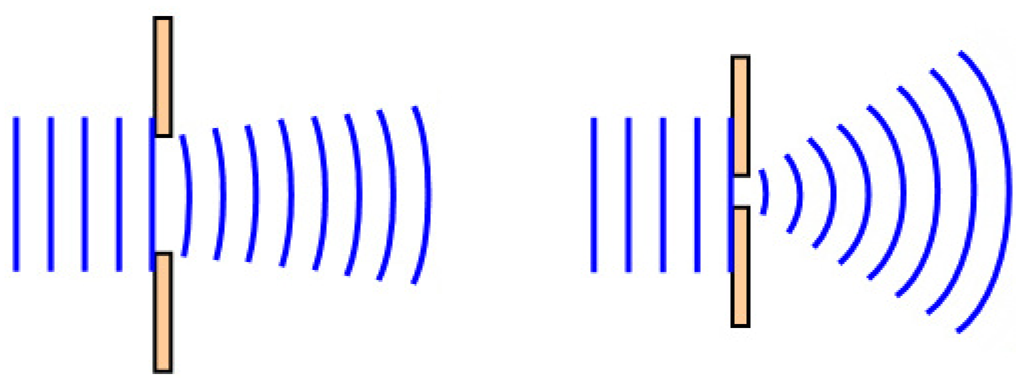
$$n_1 sin(\theta_1) = n_2 sin(\theta_2)$$

All materials have different coefficients of refraction and they are listed on the regents reference table or other sources online for you to look at. The coefficient of refraction is calculated by seeing how much light “slows down” relative to the speed of light. Light never slows down, Einstein’s theory of general relativity states as much. What actually happens is that the molecules/atoms inside the material absorb and reemit the light, giving the appearance that the light is moving slower. Think of it this way, if you’re on a highway and you travel at a constant 50 m/s, the total time you take to travel a distance of 100 m will be longer if you take breaks and continue on at 50 m/s versus if you went straight 50 m/s the entire time. The average speed would differ depending on how many breaks you took and how long those breaks were. Even though in both scenarios, the car was moving at 50 m/s. In the same way, light always travels at $3 \, x \, 10^8 \, m/s$ but the average speed of the light will differ depending on how many atoms absorb and reemit it. Therefore the coefficient of refraction is n, where n = $\frac{v}{c}$, important to note that n will ALWAYS be less than 1, so you won’t ever be confused because v will ALWAYS be less than the speed of light (Einstein’s theory of general relativity again…). Also important to note is that sin(x) function increases on the domain (0, $\frac{\pi}{2}$) which means that if the coefficient of refraction for one material is larger than the other, the angle of refraction will be less of that material to compensate for the larger coefficient of refraction. There is a critical angle that can be derived from snell's law, where any angle more will cause reflection and at the critical angle, the light ray will be normal to the boundary.

$$n_1 sin(\theta_{critical}) = n_2 sin(90)$$
$$sin\,(\theta_{critical}) = \frac{n_2}{n_1}$$
$$\theta_{critical} = sin^{-1}(\frac{n_2}{n_1})$$

Total internal reflection:
Only occurs when the ratio of the coefficient of refraction is sufficiently small. Such as diamond and air. The light tends to get trapped inside the material due to constant reflection.

Diffraction occurs when light or some other wave goes into an area through a grating. See figure 13.3 for a pictorial way of viewing this phenomenon.



The way that these waveforms produce the diffraction is related by the equation:

$$sin(\theta) = \frac{\lambda}{d}$$

θ is the angle between the central propagation and the first interference pattern. λ is the wavelength of the diffracted wave. D is the length of the slit. This should make some sense, because if you increase the slit length, the wave lengths should be larger and vice versa, this is because with a smaller slit the waves are crunched more and thus have a smaller wavelength.