Calculus 1 Cheat Sheet
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Derivations of important rules in calculus:

Product rule:

Let $y = f * g$

$ln(y) = ln(f * g)$

$ln(y) = ln(f) + ln(g)$

$\frac{d}{dx}(ln(y)) = \frac{d}{dx}(\, ln(f) + ln(g)\,)$

$\frac{y'}{y} = \frac{f'}{f} + \frac{g'}{g}$

$y' = y(\frac{f'}{f} + \frac{g'}{g})$

$y' = fg(\frac{f'}{f} + \frac{g'}{g})$

$y' = f'g + fg'$

Quotient rule:

Proof 1:

Let $y = \frac{f}{g}$

$$ln(y) = ln(\frac{f}{g})$$

$$\frac{d}{dx}(ln(y)) = \frac{d}{dx}(ln(\frac{f}{g}))$$

$$\frac{y'}{y} = \frac{d}{dx}(ln(f) - ln(g))$$

$$\frac{y'}{y} = \frac{f'}{f} - \frac{g'}{g}$$

$$y' = y(\frac{f'}{f} - \frac{g'}{g})$$

$$y' = \frac{f}{g}(\frac{f'}{f} - \frac{g'}{g})$$

$$y' = \frac{f'}{g} - \frac{g'f}{g^2}$$

$$y' = \frac{f'g - g'f}{g^2}$$

Proof 2:

Let $y = \frac{f}{g}$

$$y * g = f$$

$$y'g + yg' = f'$$

$$y'g = f' - yg'$$

$$y' = \frac{f' - yg'}{g}$$

$$y' = \frac{f' - \frac{f}{g}g'}{g}$$

$$y' = \frac{f'g - fg'}{g^2}$$

Power rule:

Let: $y = x^n$

$$ln(y) = nln(x)$$

$$\frac{y'}{y} = n\left(\frac{1}{x}\right)$$

$$\frac{y'}{y} = \frac{n}{x}$$

$$y' = y\left(\frac{n}{x}\right)$$

$$y' = \frac{nx^n}{x}$$

$$y' = nx^{n-1}$$

Natural logarithm derivative proof:

Let $y = ln(x)$

$$e^y = x$$

$$\frac{d}{dx}(e^y) = 1$$

$$e^y * y' = 1$$

$$y' = \frac{1}{e^y}$$

$$y' = \frac{1}{x}$$

Proof of the derivative of e^x :

Let y = e^x

$$ln(y) = x$$

$$\frac{y'}{y} = 1$$

$$y' = y$$

$$y' = e^x$$

Proof of the derivative of b^x :

Let y = b^x

$$ln(y) = xln(b)$$

$$\frac{y'}{y} = ln(b)$$

$$y' = y\ ln(b)$$

$$y' = b^x ln(b)$$

Plug in b = e for special surprise

Proof of the derivative of tan(x) :

Let $y = tan(x)$

$$y = \frac{sin(x)}{cos(x)}$$

Quotient rule

$$y' = \frac{cos^2(x) + sin^2(x)}{cos^2(x)}$$

$$y' = \frac{1}{cos^2(x)}$$

$$y' = sec^2(x)$$

Proof of the derivative of cot(x) :

Let $y = cot(x)$

$$y = \frac{cos(x)}{sin(x)}$$

Quotient rule

$$y' = \frac{-sin^2(x) - cos^2(x)}{sin^2(x)}$$

$$y' = \frac{-1(sin^2(x) + cos^2(x))}{sin^2(x)}$$

$$y' = \frac{-1}{sin^2(x)}$$

$$y' = -csc^2(x)$$

Inverse Function Derivative Theorem

Let $y = f^{-1}(x)$

$$f(y) = x$$
$$\frac{d}{dx}(f(y)) = 1$$
$$f'(y) * y' = 1$$
$$y' = \frac{1}{f'(y)}$$

Derivative properties:

$$\frac{dy}{dx} = y' = first\ derivative\ of\ y\ with\ respect\ to\ x$$

$\frac{d^2y}{dx^2} = y'' = second\ derivative\ of\ y\ with\ respect\ to\ x$

$\frac{d}{dx}(f(g)) = f'(g) * g'$

$\frac{d}{dx}(f * g) = f'g + fg'$

$\frac{d}{dx}(\frac{f}{g}) = \frac{f'g - fg'}{g^2}$

If $g = -y, \frac{dg}{dx} = -\frac{dy}{dx}$

If $g = ky, \frac{dg}{dx} = k\frac{dy}{dx}$

If $y = f^{-1}(x), \frac{dy}{dx} = \frac{1}{f'(y)}$ inverse function, not reciprocal

Memorized derivatives:

$\frac{d}{dx}(sin(x)) = cos(x)$

$\frac{d}{dx}(cos(x)) = -sin(x)$

$\frac{d}{dx}(-sin(x)) = -\frac{d}{dx}(sin(x)) = -cos(x)$

$\frac{d}{dx}(-cos(x)) = -\frac{d}{dx}(cos(x)) = -1(-sin(x)) = sin(x)$

$\frac{d}{dx}(ln(x)) = \frac{1}{x}$

$\frac{d}{dx}(tan(x)) = sec^2(x)$

$\frac{d}{dx}(sec(x)) = sec(x)tan(x)$

$$\frac{d}{dx}(csc(x)) = -csc(x)cot(x)$$

$$\frac{d}{dx}(cot(x)) = -csc^2(x)$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(b^x) = ln(b)\, b^x$$

$$\frac{d}{dx}(arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(arctan(x)) = \frac{1}{1+x^2}$$

Memorized integrals:

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x}\, dx = ln(x) + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = arcsin(x) + C$$

$$\int sin(x)\, dx = -cos(x) + C$$

$$\int cos(x)\,dx \;=\; sin(x) \;+\; C$$

$$\int - sin(x)\,dx \;=\; cos(x) \;+\; C$$

$$\int - cos(x)\,dx \;=\; -sin(x) \;+\; C$$

$$\int sec^2(x)\,dx \;=\; tan(x) \;+\; C$$

$$\int csc^2(x)\,dx \;=\; -\; cot(x) \;+\; C$$

$$\int sec(x)tan(x)\,dx \;=\; sec(x) \;+\; C$$

$$\int csc(x)cot(x)\,dx \;=\; -csc(x) \;+\; C$$

$$\int tan(x)\,dx \;=\; ln(sin(x)) \;+\; C$$

$$\int cot(x)\,dx \;=\; -ln(cos(x)) \;+\; C$$

$$\int sin(kx)\,dx \;=\; -\;\tfrac{1}{k}cos(kx) \;+\; C$$

$$\int cos(kx)\,dx \;=\; \tfrac{1}{k}sin(kx) \;+\; C$$

$$\int - sin(kx)\,dx \;=\; \tfrac{1}{k}cos(kx) \;+\; C$$

$$\int - cos(kx)\, dx \;=\; -\frac{1}{k}\, sin(kx) \;+\; C$$

$$\int tan(kx)\, dx \;=\; \frac{1}{k}(ln(sin(x)) \;+\; C$$

$$\int cot(kx)\, dx \;=\; -\frac{1}{k}(ln(cos(x)) \;+\; C$$

$$\int e^{x}\, dx \;=\; e^{x} \;+\; C$$

$$\int e^{-kx}\, dx \;=\; -\frac{1}{k}\,(e^{-kx}) \;+\; C$$

$$\int e^{kx}\, dx \;=\; \frac{1}{k}\, e^{kx} \;+\; C$$

$$\int sec(x)\, dx \;=\; ln(sec(x) \;+\; tan(x)\,) \;+\; C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx \;= arcsin(\frac{x}{a}) \;+\; C$$

$$\int \frac{1}{a^2 + x^2}\, dx \;=\; \frac{1}{a}\, arctan(\frac{x}{a}) \;+\; C$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx \;=\; arccos(\frac{x}{a}) \;+\; C$$

Integral properties:

$\int \frac{d}{dx}(f(x))\,dx = f(x)$ also known as the second fundamental theorem of calculus (FTC)

$\int_a^b f(x)\,dx = F(b) - F(a)$ *where F is the antiderivative of f*, $F' = f$ also known as the first FTC.

$\int_{-a}^{a} f(x)\,dx = 0$, *if f(x) is an odd function* due to origin symmetry

$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, *if f(x) is an even function* due to y-axis symmetry

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b \frac{d}{dx}(f(x))\,dx = f(b) - f(a)$$

$$\int - f(x)\,dx = -\int f(x)\,dx$$

$$\int k\,f(x)\,dx = k\int f(x)\,dx$$

$$-\int_a^b f(x)\,dx = \int_b^a f(x)\,dx$$

$$\frac{d}{dx}\left(\int_k^{g(x)} f(t)\,dt\right) = f(g(x)) * g'(x)$$

$$\int \frac{f'(x)}{f(x)}\,dx = ln(f(x)) + C$$

$V = \pi\int_a^b (r(x))^2\,dx$ assuming you could revolve the function around to make a disc, not a washer (there are no functions underneath it)

$V = \pi\int_a^b (R(x))^2 - (r(x))^2\,dx$ assuming there is a lower function hence r(x)'s inclusion.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\left(\sum_{k=1}^{n} \left(f(x_k) * \frac{b-a}{n}\right)\right)$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\left(\sum_{k=1}^{n} \left(f(x_k)\,\Delta x_k\right)\right)$$

Hard derivations (not necessary to know)

Proof of the derivative of sin(x)

Let $y = sin(x)$

$$arcsin(y) = x$$
$$\frac{d}{dx}(arcsin(y)) = 1$$

$$\frac{y'}{\sqrt{1-y^2}} = 1$$

$$y' = \sqrt{1-y^2}$$

$$y' = \sqrt{1-sin^2(x)}$$

Pythagorean identity

$$y' = \sqrt{cos^2(x)}$$

$$y' = |cos(x)|$$

Is it +cos(x) or -cos(x)?

Suppose $y' = -cos(x)$

That would suggest that $sin(x)$ is decreasing from $0 \; to \; \frac{\pi}{2}$

This is because y' evaluates to a negative number on that domain. $sin(x)$ is not decreasing from $0\ to\ \frac{\pi}{2}$ because $sin(\frac{\pi}{2})$ is greater than $sin(0)$. Therefore we must reject the negative solution.

$$y' = cos(x)$$

Proof of the derivative of cos(x)

Let $y = cos(x)$

$$arccos(y) = x$$

$$\frac{d}{dx}(arccos(y)) = 1$$

$$\frac{-y'}{\sqrt{1-y^2}} = 1$$

$$-y' = \sqrt{1-y^2}$$

$$y' = -\sqrt{1-y^2}$$

$$y' = -\sqrt{1-cos^2(x)}$$

Pythagorean identity

$$y' = -\sqrt{sin^2(x)}$$

$$y' = -|sin(x)|$$

Is it +sin(x) or -sin(x)?

cos(x) decreases from $0 \ to \ \frac{\pi}{2}$
Therefore we need to have positive sin(x) because that guarantees that cos(x) will decrease from $0 \ to \ \frac{\pi}{2}$

$$y' = -sin(x)$$

Proof of the derivative of csc(x)

Let $y = csc(x)$

$$y = \frac{1}{sin(x)}$$

$$y * sin(x) = 1$$

$$\frac{d}{dx}(y * sin(x)) = 0$$

Product rule

$y'sin(x) + ycos(x) = 0$

$y'sin(x) = -ycos(x)$

$y' = -y * \frac{cos(x)}{sin(x)}$

$y' = -y * cot(x)$

$y' = -csc(x)cot(x)$

Proof of the derivative of sec(x)

Let $y = sec(x)$

$y = \frac{1}{cos(x)}$

$y * cos(x) = 1$

Product rule

$\frac{d}{dx}(y * cos(x)) = 0$

$y'cos(x) - ysin(x) = 0$

$$y'cos(x) = ysin(x)$$

$$y' = y * \frac{sin(x)}{cos(x)}$$

$$y' = y\ tan(x)$$

$$y' = sec(x)tan(x)$$

Proof of the derivative of arcsin(x)

Let $y = arcsin(x)$

$$sin(y) = x$$

$$\frac{d}{dx}(sin(y)\ ) = 1$$

$$cos(y) * y' = 1$$

$$y' = \frac{1}{cos(y)}$$

$$y' = sec(y)$$

Recall that $sin(y) = x$, in general $sin(\theta) = \frac{Opposite}{Hypotenuse}$

This implies that $sin(y) = \frac{x}{1}$

Now we know the opposite side is x, the hypotenuse is 1

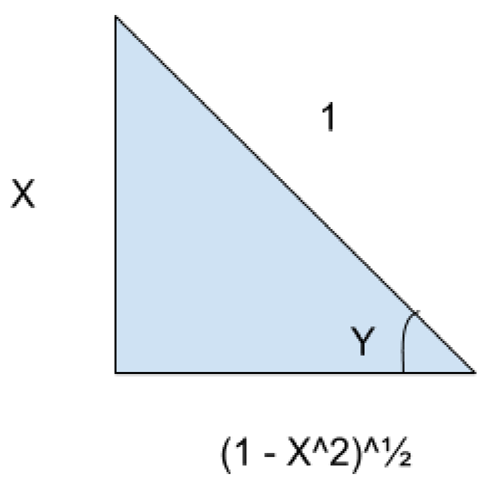
Applying pythagorean theorem, we get the adjacent side is

$$x^2 + Adjacent^2 = 1$$

$$Adjacent^2 = 1 - x^2$$

$$Adjacent = \sqrt{1 - x^2}$$

Thus the following triangle is constructed:



We know that $y' = sec(y)$

$$sec(y) = \frac{Hypotenuse}{Adjacent} = \frac{1}{\sqrt{1-x^2}}$$

Therefore we know that

$$y' = \frac{1}{\sqrt{1-x^2}}$$

The same process can be used to prove all of the inverse trigonometric functions :)